

Waves in the Atmosphere and Oceans

Restoring Force

- ❑ Conservation of potential temperature in the presence of positive static stability
→ internal gravity waves
- ❑ Conservation of potential vorticity in the presence of a mean gradient of potential vorticity → Rossby waves

- **External gravity wave** (Shallow-water gravity wave)
- **Internal gravity (buoyancy) wave**
- **Inertial-gravity wave**: Gravity waves that have a large enough wavelength to be affected by the earth's rotation.
- **Rossby Wave**: Wavy motions results from the conservation of potential vorticity.
- **Kelvin wave**: It is a wave in the ocean or atmosphere that balances the Coriolis force against a topographic boundary such as a coastline, or a waveguide such as the equator. Kelvin wave is non-dispersive.



Lecture 6: Adjustment under Gravity in a Non-Rotating System



- Overview of Gravity waves
- Surface Gravity Waves
- “Shallow” Water
- Shallow-Water Model
- Dispersion



Chapter Five **Adjustment under Gravity in a Nonrotating System**

5.1	Introduction: Adjustment to Equilibrium	95
5.2	Perturbations from the Rest State for a Homogenous Inviscid Fluid	99
5.3	Surface Gravity Waves	101
5.4	Dispersion	104
5.5	Short-Wave and Long-Wave Approximations	106
5.6	Shallow-Water Equations Derived Using the Hydrostatic Approximation	107
5.7	Energetics of Shallow-Water Motion	111
5.8	Seiches and Tides in Channels and Gulfs	112



Goals of this Chapter

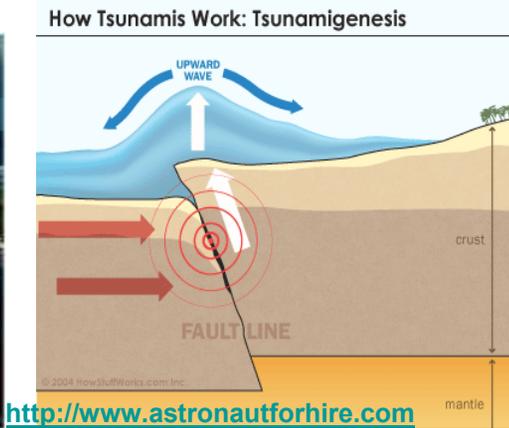
- This chapter marks the beginning of more detailed study of the way the atmosphere-ocean system *tends to adjust to equilibrium*.
- The adjustment processes are most easily understood in the absence of driving forces. Suppose, for instance, that the sun is "switched off," leaving the atmosphere and ocean with some non-equilibrium distribution of properties.
- How will they respond to the gravitational restoring force?
- Presumably there will be an adjustment to some sort of equilibrium. If so, what is the nature of the equilibrium?
- In this chapter, complications due to the rotation and shape of the earth will be ignored and only small departures from the hydrostatic equilibrium will be considered.
- The nature of the adjustment processes will be found by deduction from the equations of motion



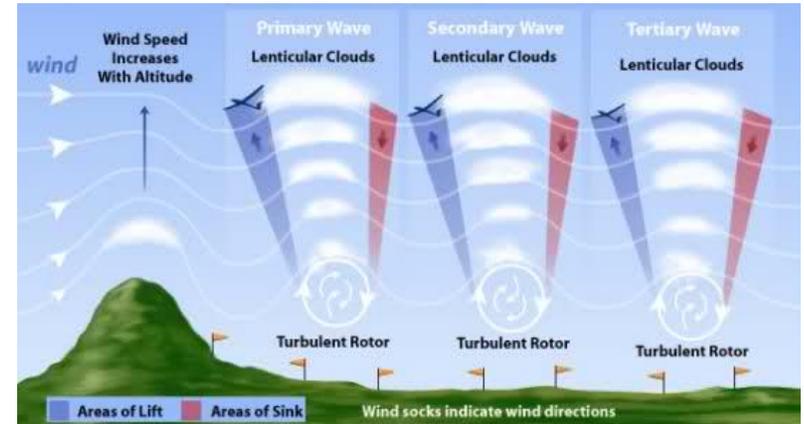
Gravity Waves



Bernie Baker



<http://skywarn256.wordpress.com>



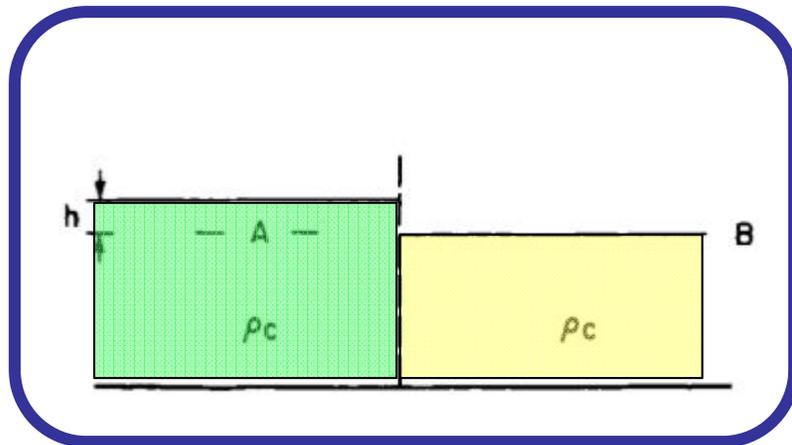
- Gravity waves are waves generated in a fluid medium or at the interface between two media (e.g., the atmosphere and the ocean) which has the restoring force of gravity or buoyancy.
- When a fluid element is displaced on an interface or internally to a region with a different density, gravity tries to restore the parcel toward equilibrium resulting in an oscillation about the equilibrium state or wave orbit.
- Gravity waves on an air-sea interface are called surface gravity waves or surface waves while internal gravity waves are called internal waves.



Adjustment Under Gravity in a Non-Rotating System

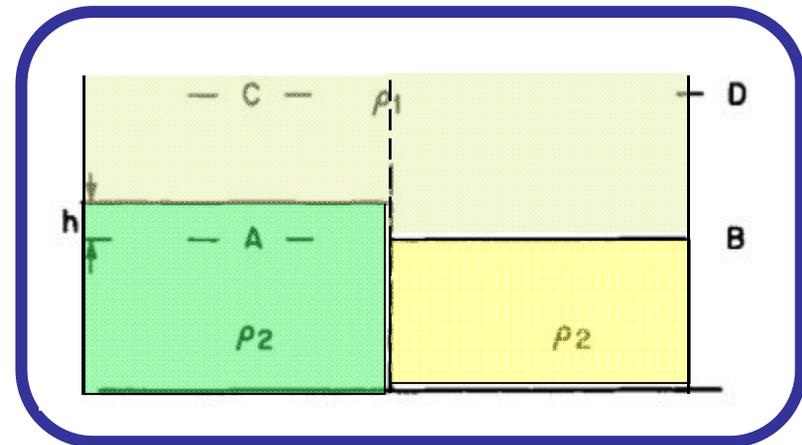
External Gravity Waves

adjustment of a homogeneous fluid with a free surface

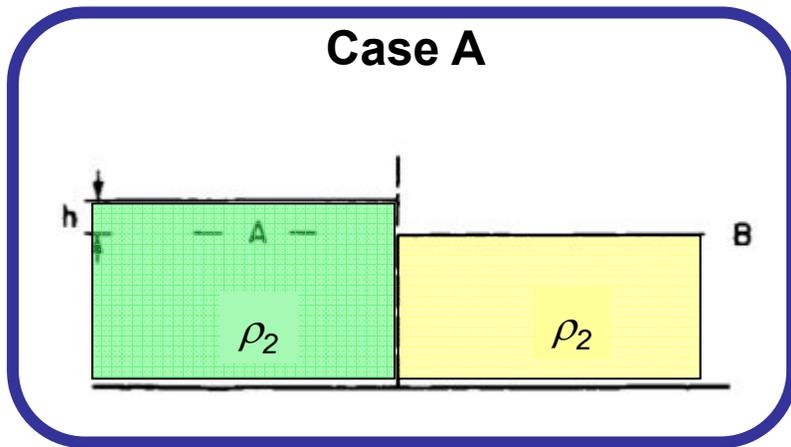


Internal Gravity Waves

adjustment of a density-stratified fluid

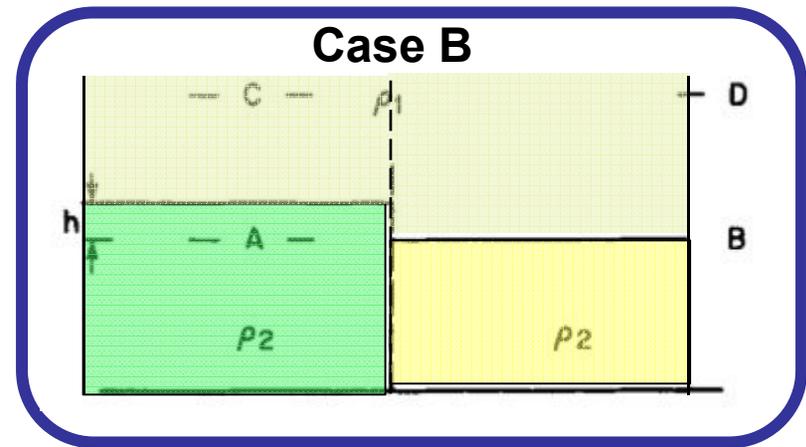


Reduced Gravity



Pressure difference between A and B:

$$\Delta P = \rho_2 * g * h$$



Pressure difference between A and B:

$$\Delta P = (\rho_2 - \rho_1) * g * h$$

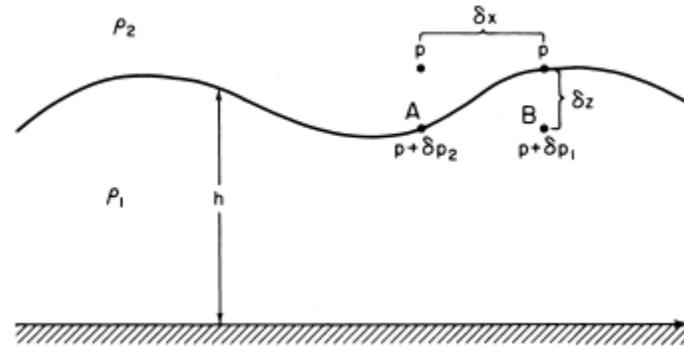
The adjustment process in Case B is exactly the same as in the Case A, except the gravitational acceleration is reduced to a value g' , where

$$g' = g(\rho_2 - \rho_1) / \rho_2.$$

buoyancy force =
density difference * g



A One-Layer Fluid System



We assume that the motion is two dimensional in the x, z plane.

$$\delta\rho = \rho_1 - \rho_2.$$

Momentum Equation

$$P_A = p + \delta p_1 = p + \rho_1 g \delta z = p + \rho_1 g \left(\frac{\partial \hat{h}}{\partial x} \right) \delta x$$

$$P_B = p + \delta p_2 = p + \rho_2 g \delta z = p + \rho_2 g \left(\frac{\partial \hat{h}}{\partial x} \right) \delta x$$

$$\square PG = \lim_{\delta x \rightarrow 0} \left[\frac{(p + \delta p_1) - (p + \delta p_2)}{\delta x} \right] = g \delta \rho \frac{\partial \hat{h}}{\partial x}$$

□ Momentum equation:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = - \frac{g \delta \rho}{\rho_1} \frac{\partial \hat{h}}{\partial x}$$

Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

$$\rightarrow w(\hat{h}) - w(0) = -\hat{h} \left(\frac{\partial u}{\partial x} \right) \begin{cases} w(\hat{h}) = \frac{D\hat{h}}{Dt} = \frac{\partial \hat{h}}{\partial t} + u \frac{\partial \hat{h}}{\partial x} \\ w(0) = 0 \end{cases}$$

$$\rightarrow \frac{\partial \hat{h}}{\partial t} + u \frac{\partial \hat{h}}{\partial x} + \hat{h} \frac{\partial u}{\partial x} = \frac{\partial \hat{h}}{\partial t} + \frac{\partial}{\partial x} (\hat{h} u) = 0$$

$$u = \bar{u} + u', \quad \hat{h} = H + \hat{h}'$$

applying the perturbation method

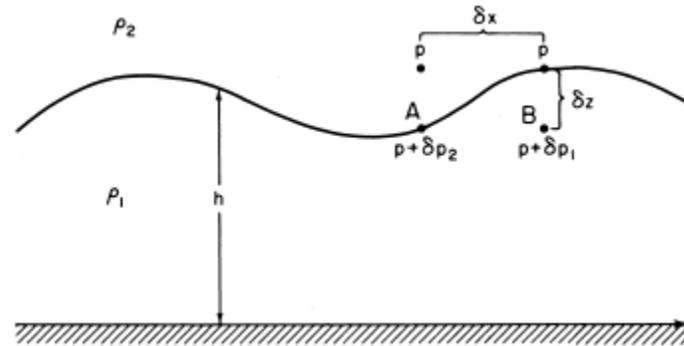
$$\begin{aligned} \frac{\partial u'}{\partial t} + \bar{u} \frac{\partial u'}{\partial x} + \frac{g \delta \rho}{\rho_1} \frac{\partial \hat{h}'}{\partial x} &= 0 \\ \frac{\partial \hat{h}'}{\partial t} + \bar{u} \frac{\partial \hat{h}'}{\partial x} + H \frac{\partial u'}{\partial x} &= 0 \end{aligned}$$

→

$$\left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} \right)^2 \hat{h}' - \frac{g H \delta \rho}{\rho_1} \frac{\partial^2 \hat{h}'}{\partial x^2} = 0$$



Shallow Water Gravity Wave



We assume that the motion is two dimensional in the x, z plane.

$$\delta\rho = \rho_1 - \rho_2.$$

Governing Equation

$$\left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} \right)^2 \eta' - \frac{gH\delta\rho}{\rho_1} \frac{\partial^2 \eta'}{\partial x^2} = 0$$

$$\eta' = A \exp[ik(x - ct)]$$

$$c = \bar{u} \pm (gH\delta\rho/\rho_1)^{1/2}$$

$$\delta\rho \approx \rho_1 \text{ (e.g., air and water)}$$

$$c = \bar{u} \pm \sqrt{gH}$$

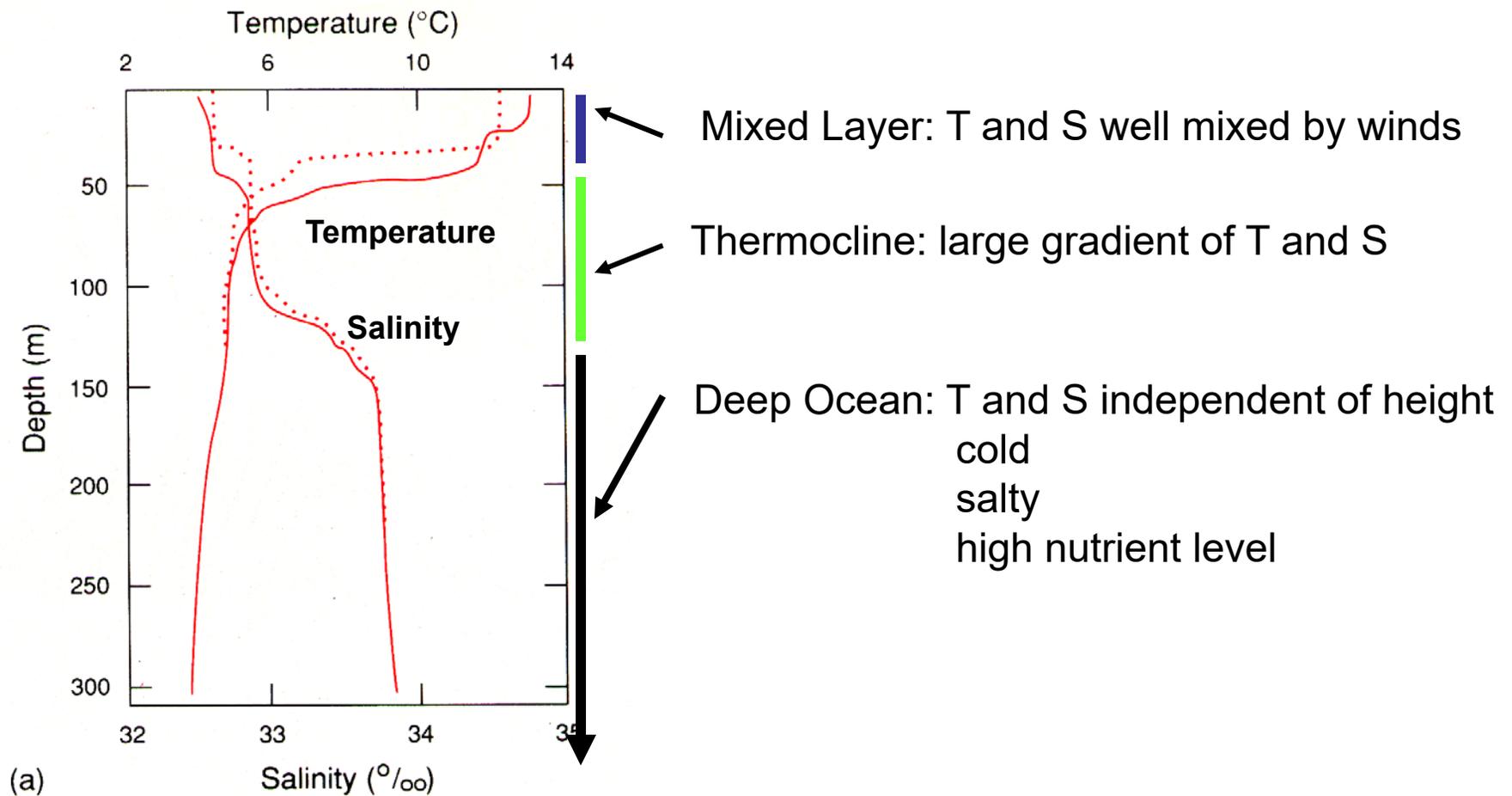
- The shallower the water, the slower the wave.
- Shallow water gravity waves are non-dispersive.

- ❑ Shallow water gravity waves may also occur at thermocline where the surface water is separated from the deep water. *(These waves can also referred to as the internal gravity waves).*
- ❑ If the density changes by an amount $\delta\rho/\rho_1 \approx 0.01$, across the thermocline, then the wave speed for waves traveling along the thermocline will be only one-tenth of the surface wave speed for a fluid of the same depth.

Shallow water wave speed $\approx 200 \text{ ms}^{-1}$ for an ocean depth of 4km



Vertical Structure of Ocean



(from *Climate System Modeling*)

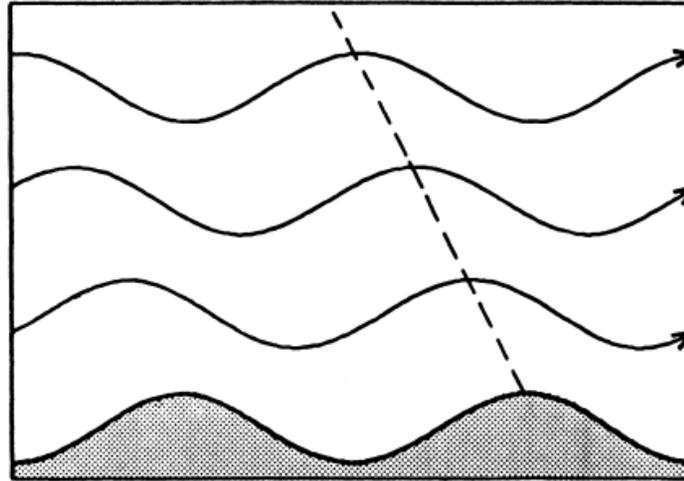


Shallow and Deep Water

- “Shallow” in this lecture means that the depth of the fluid layer is small compared with the horizontal scale of the perturbation, i.e., the horizontal scale is large compared with the vertical scale.
- Shallow water gravity waves are the “long wave approximation” end of gravity waves.
- Deep water gravity waves are the “short wave approximation” end of gravity waves.
- Deep water gravity waves are not important to large-scale motions in the oceans.



Internal Gravity (Buoyancy) Waves



- ❑ In a fluid, such as the ocean, which is bounded both above and below, gravity waves propagate primarily in the horizontal plane since vertically traveling waves are reflected from the boundaries to form standing waves.
- ❑ In a fluid that has no upper boundary, such as the atmosphere, gravity waves may propagate vertically as well as horizontally. In vertically propagating waves the phase is a function of height. Such waves are referred to as *internal waves*.
- ❑ *Although internal gravity waves are not generally of great importance for synoptic-scale weather forecasting (and indeed are nonexistent in the filtered quasi-geostrophic models), they can be important in mesoscale motions.*
- ❑ For example, they are responsible for the occurrence of mountain *lee waves*. *They also are believed to be an important mechanism for transporting energy and momentum into the middle atmosphere, and are often associated with the formation of clear air turbulence (CAT).*



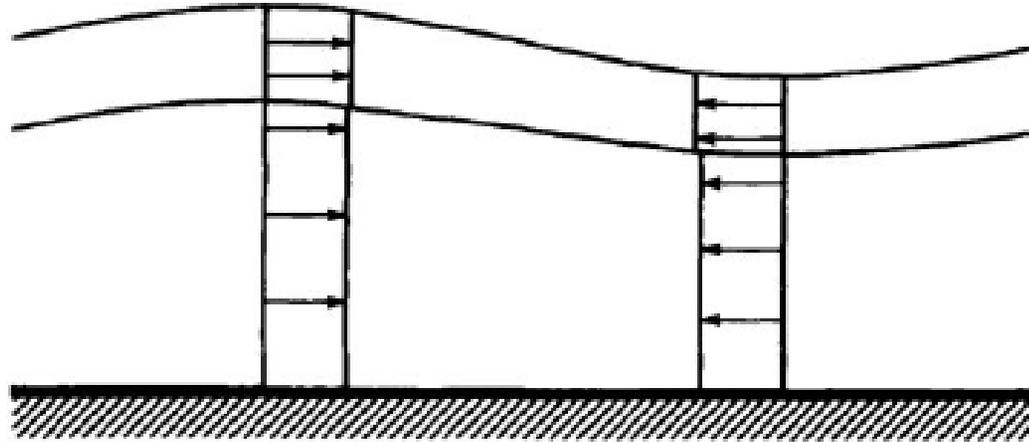
Quasi-Geostrophic Approximation

$$\frac{\partial \zeta_g}{\partial t} + \vec{V}_g \cdot \nabla \zeta_g + \beta v_g = -f \nabla \cdot \vec{V}$$

- Quasi-geostrophic approximation use the geostrophic wind for the actual wind everywhere *except* when computing divergence.
- The Q-G approximation eliminates both sound and gravity waves as solutions to the equations of motion.



Lecture 7: Adjustment under Gravity of a Density-Stratified Fluid



- Normal Mode & Equivalent Depth
- Rigid Lid Approximation
- Boussinesq Approximation
- Buoyancy (Brunt-Väisälä) Frequency
- Dispersion of internal gravity waves

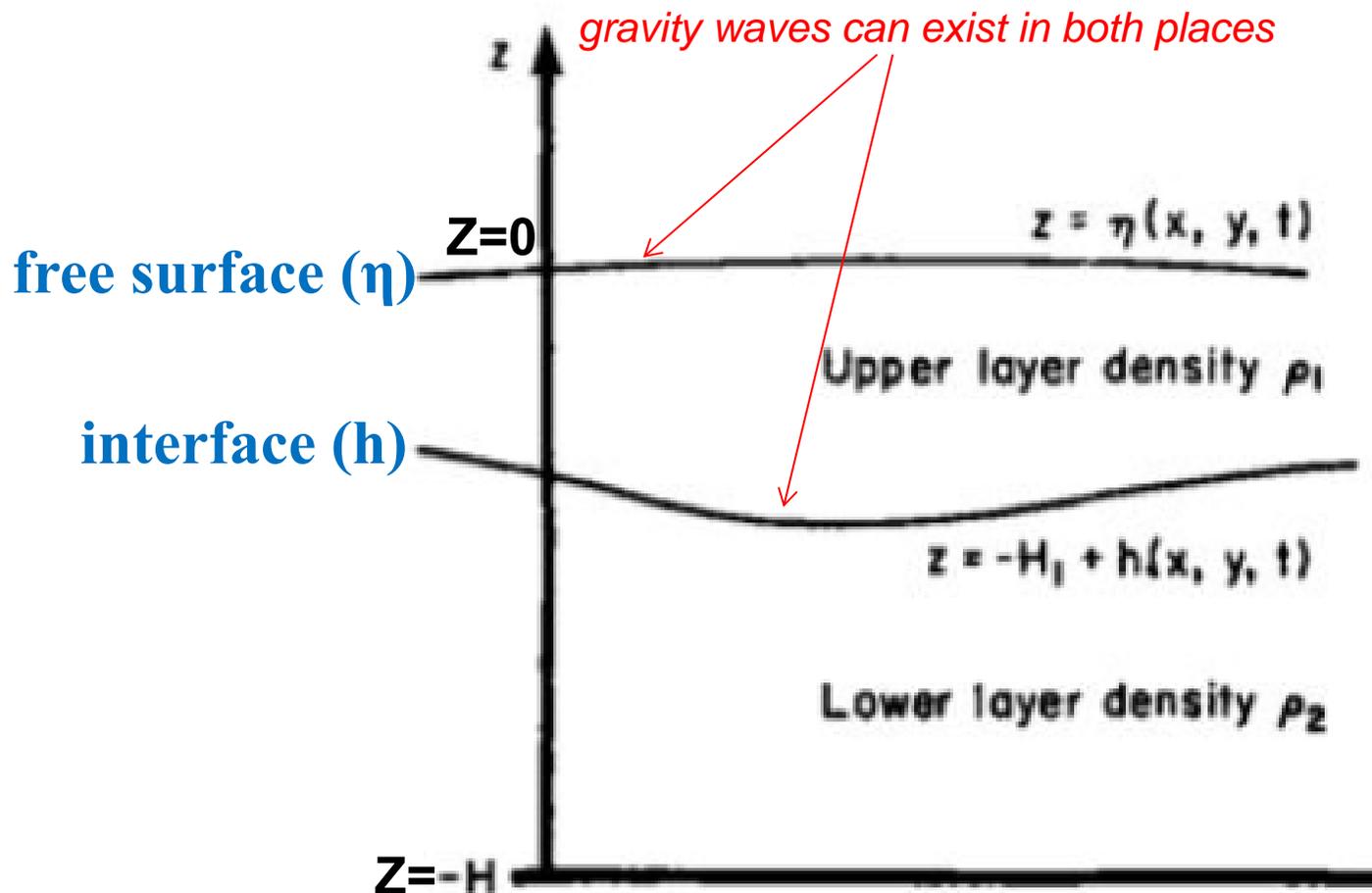


Main Purpose of This Lecture

- As an introduction to the effects of stratification, the case of two superposed shallow layers, each of uniform density, is considered.
- In reality, both the atmosphere and ocean are continuously stratified.
- This serves to introduce the concepts of *barotropic* and *baroclinic* modes.
- This also serves to introduce two widely used approximations: the *rigid lid approximation* and the *Boussinesq approximation*.



Two Fluids of Different Density



Fluid 1

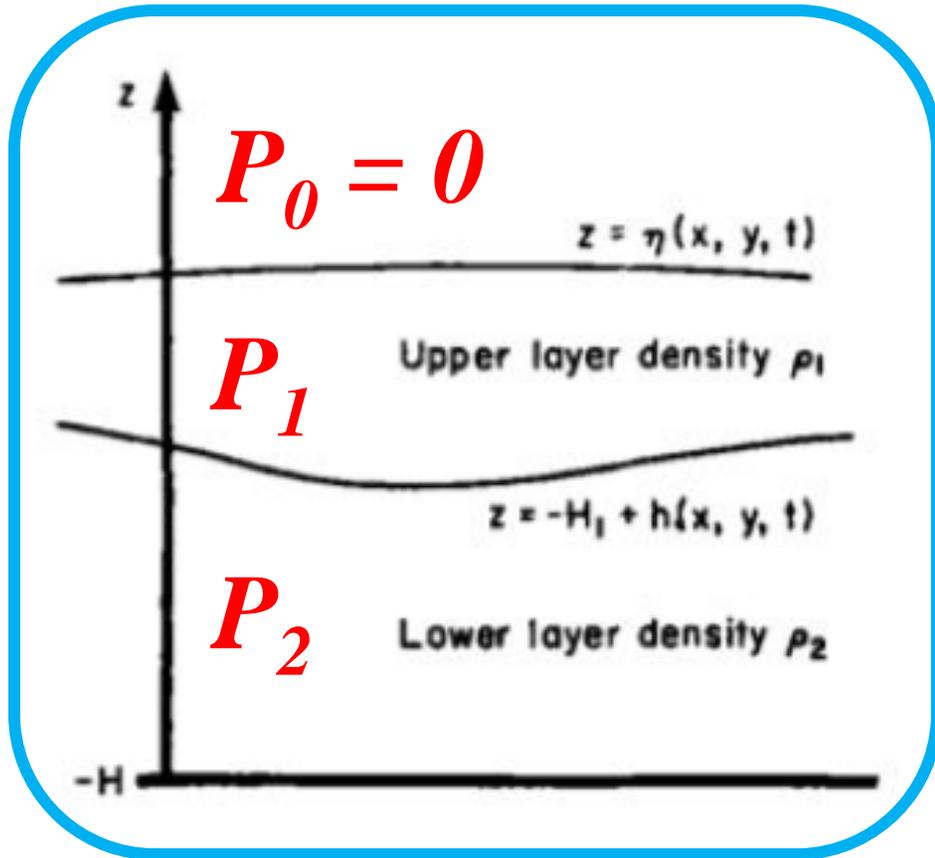
density = ρ_1
mean depth = H_1
velocity = U_1, V_1

Fluid 2

density = ρ_2
mean depth = H_2
velocity = U_2, V_2



Two Fluids: *Layer 1* ($-H_1 + h < z < \eta$)



$$p_1 = \rho_1 g(\eta - z)$$

□ Momentum Equations

$$\partial u_1 / \partial t = -g \partial \eta / \partial x,$$

$$\partial v_1 / \partial t = -g \partial \eta / \partial y,$$

□ Continuity Equation

$$\partial(\eta + H_1 - h) / \partial t +$$

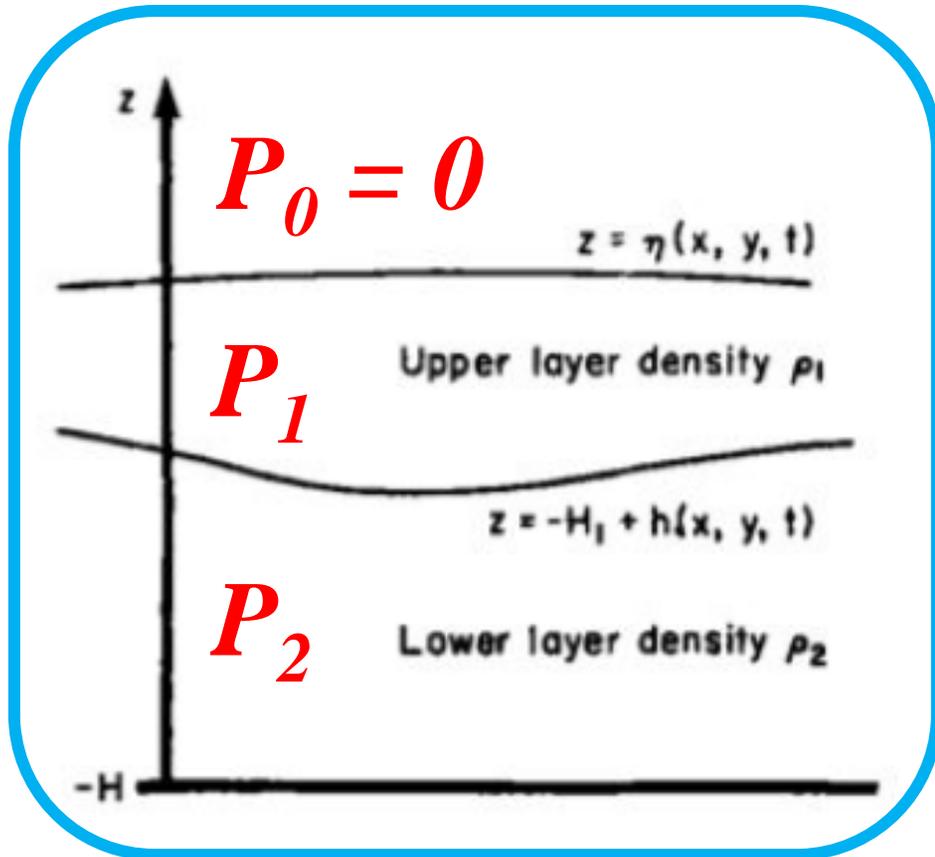
$$H_1(\partial u_1 / \partial x + \partial v_1 / \partial y) = 0.$$

□ Taking time derivative of the continuity equation:

$$\frac{\partial^2}{\partial t^2} (\eta - h) = H_1 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) g \eta \equiv g H_1 \nabla^2 \eta$$

Two Fluids: *Layer 2* ($z < -H_1 + h$)

$$p_2 = \rho_1 g(\eta + H_1 - h) + \rho_2 g(-H_1 + h - z).$$



□ Momentum Equations

$$\frac{\partial u_2}{\partial t} = -\frac{\rho_1}{\rho_2} g \frac{\partial \eta}{\partial x} - g' \frac{\partial h}{\partial x}$$

$$\frac{\partial v_2}{\partial t} = -\frac{\rho_1}{\rho_2} g \frac{\partial \eta}{\partial y} - g' \frac{\partial h}{\partial y}$$

$$g' = g(\rho_2 - \rho_1)/\rho_2$$

= reduced gravity

□ Continuity Equation

$$\partial h / \partial t + H_2 (\partial u_2 / \partial x + \partial v_2 / \partial y) = 0.$$

□ Taking time derivative of the continuity

$$\frac{\partial^2 h}{\partial t^2} = H_2 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left(\frac{\rho_1}{\rho_2} g \eta + g' h \right) = H_2 \nabla^2 (g \eta - g' \eta + g' h),$$

Adjustments of the Two-Fluid System

- The adjustments in the two-layer fluid system are governed by:

$$\frac{\partial^2}{\partial t^2}(\eta - h) = H_1 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) g\eta \equiv gH_1 \nabla^2 \eta$$

$$\frac{\partial^2 h}{\partial t^2} = H_2 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left(\frac{\rho_1}{\rho_2} g\eta + g'h \right) = H_2 \nabla^2 (g\eta - g'\eta + g'h),$$

- Combined these two equations will result in a fourth-order equation, which is difficult to solve.
- This problem can be greatly simplified by looking for solutions which η and h are proportional:

$$h(x, y, t) = \mu\eta(x, y, t),$$

- The governing equations will both reduced to this form:

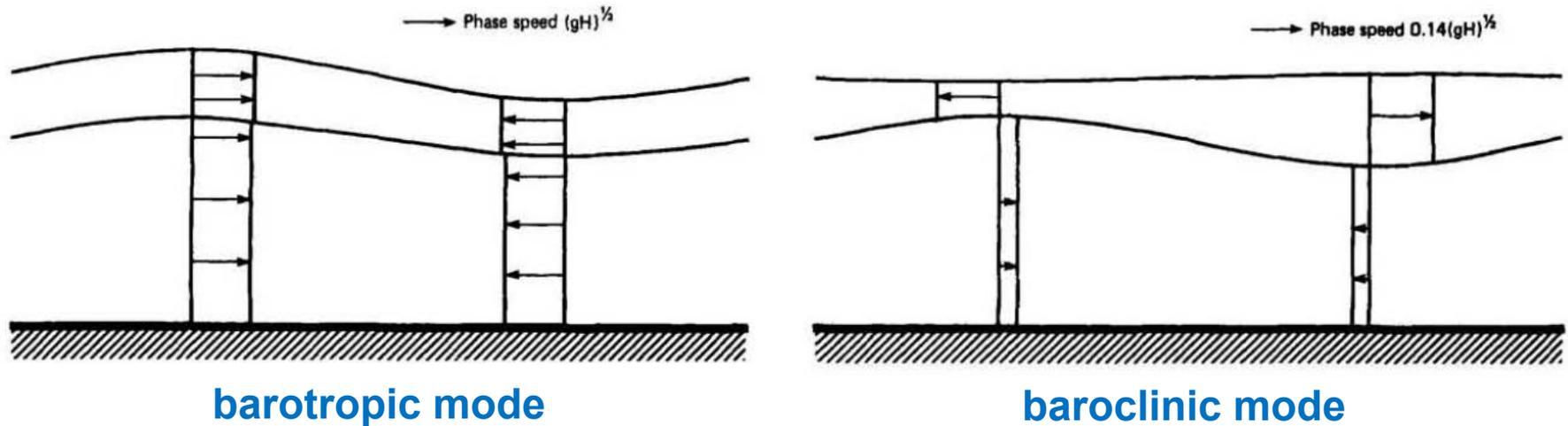
$$\frac{\partial^2 \eta}{\partial t^2} = c_e^2 \nabla^2 \eta, \quad \text{provided that}$$

$$gH_1/(1 - \mu) = \mu^{-1}(g - g'(1 - \mu))H_2 = c_e^2.$$

There are two values of μ (and hence two values of c_e) that satisfy this equation.

→ The motions corresponding to these particular vales are called normal modes of oscillation.

Normal Modes



- The motions corresponding to these particular values of c_e or μ are called *normal modes* of oscillation.
- In a system consisting of n layers of different density, there are n normal modes corresponding to the n degrees of freedom.
- A continuously stratified fluid corresponding to an infinite number of layers, and so there is an infinite set of modes.



Structures of the Normal Modes

- The structures of the normal modes can be obtained by solving this equation (from previous slide):

$$c_e^4 - gHc_e^2 + gg'H_1H_2 = 0,$$

$$H = H_1 + H_2$$

- Or, we can use the concept of the one-layer shallow water model, where the phase speed (c) of the gravity wave is related to the depth of the shallow water (H):

$$c = \sqrt{gH}$$

- Using this concept, we can assume each of the normal mode behaves like the one-layer shallow water with a “equivalent depth” of H_e :

$$c_e^2 = gH_e.$$

$$gH_e^2 - gHH_e + g'H_1H_2 = 0.$$

$$c_0^2 = gH(1 - g'H_1H_2/gH^2 \dots),$$

$$\eta/h \approx H/H_2, \quad u_2/u_1 = 1 - g'H_1/gH \dots$$

Solution 1
(barotropic mode)

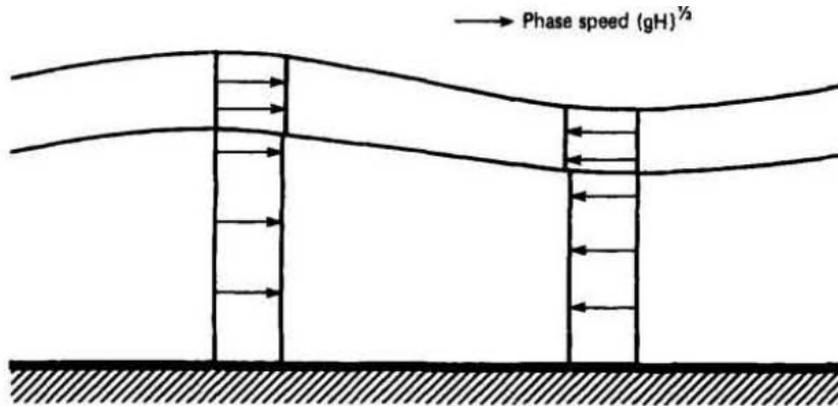
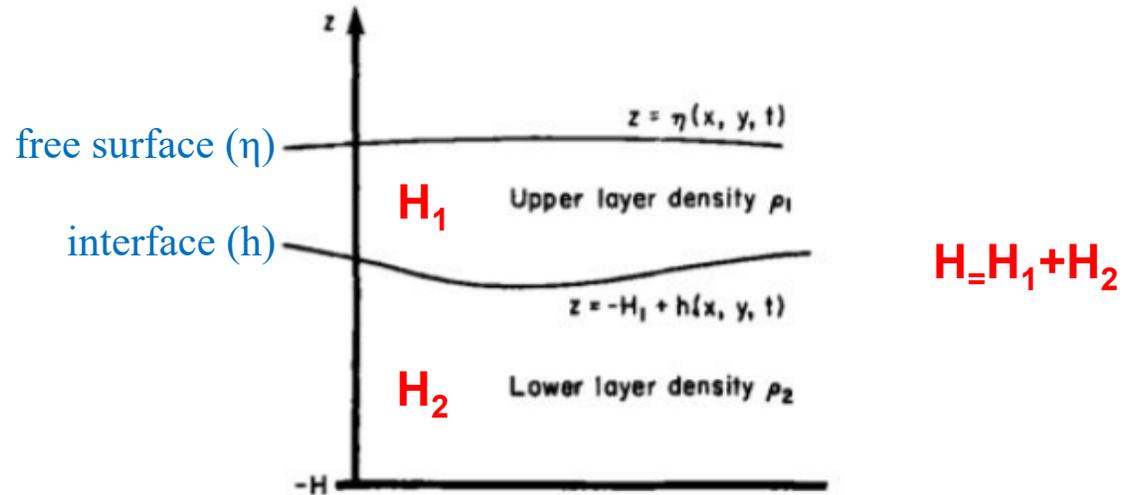
$$c_1^2 = (g'H_1H_2/H)(1 + g'H_1H_2/gH^2 \dots),$$

$$\eta/h \approx -g'H_2/gH, \quad u_2/u_1 \approx -H_1/H_2.$$

Solution 2
(baroclinic mode)



Structures of the Normal Modes

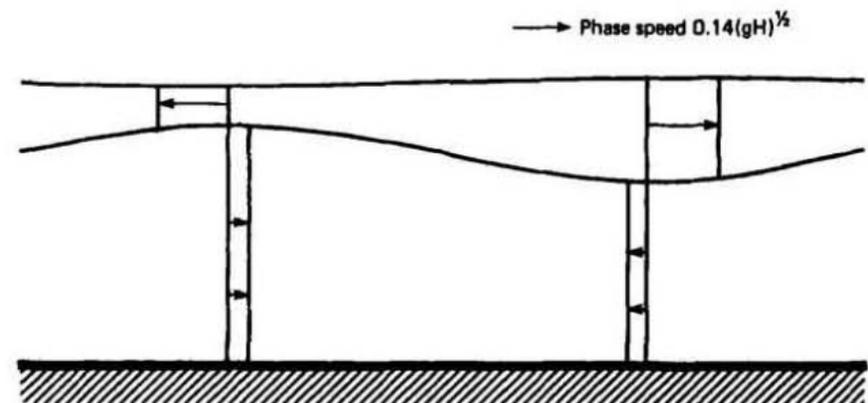


Barotropic mode

$$c_0^2 = gH(1 - g'H_1H_2/gH^2 \dots),$$

$$\eta/h \approx H/H_2, \quad u_2/u_1 = 1 - g'H_1/gH \dots$$

$\eta > h$; u_2 & u_1 of same signs



Baroclinic mode

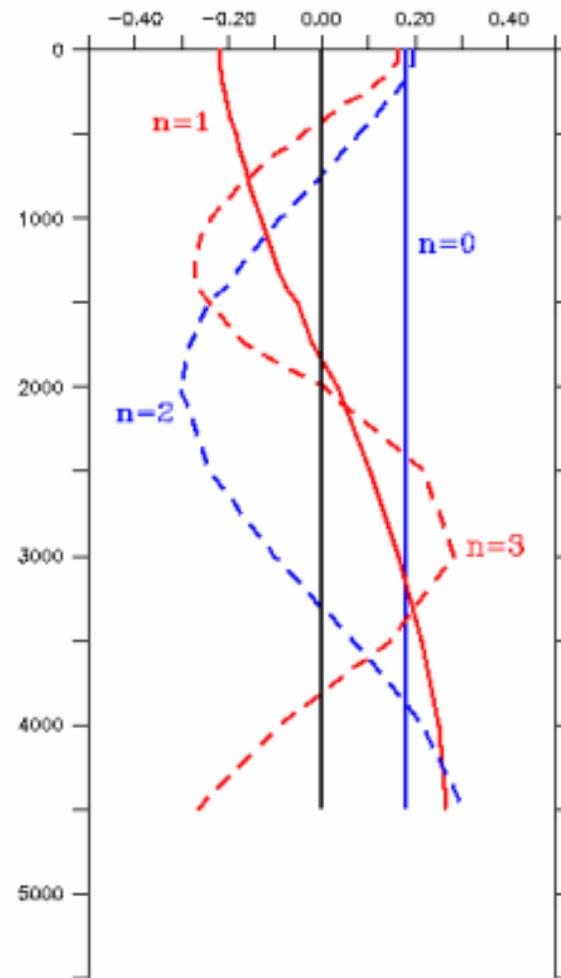
$$c_1^2 = (g'H_1H_2/H)(1 + g'H_1H_2/gH^2 \dots),$$

$$\eta/h \approx -g'H_2/gH, \quad u_2/u_1 \approx -H_1/H_2.$$

$\eta < h$; u_2 & u_1 of opposite signs

Example

first four modes (0,1,2 & 3) 63E, 18S



Equivalent Depth (H_e)

- An N-layer fluid will have one barotropic mode and (N-1) baroclinic modes of gravity waves, each of which has its own equivalent depth.
- Once the equivalent depth is known, we know the dispersion relation of that mode of gravity wave and we know how fast/slow that gravity wave propagates.

$$c_e^2 = gH_e.$$

For the barotropic mode: $H_e = H$

For the baroclinic mode: $H_e = g'/g * H_1 H_2 / H$

- For a continuously stratified fluid, it has an infinite number of modes, but not all the modes are important. We only need to identify the major baroclinic modes and to find out their equivalent depths.



An example of Equivalent Atmosphere

- Equivalent depth for an incompressible atmosphere

$$h_0 = \frac{N^2 H^2}{\pi^2 g}.$$

- Rossby radius of deformation

For a barotropic ocean: $L_R \equiv \frac{(gD)^{1/2}}{f}$

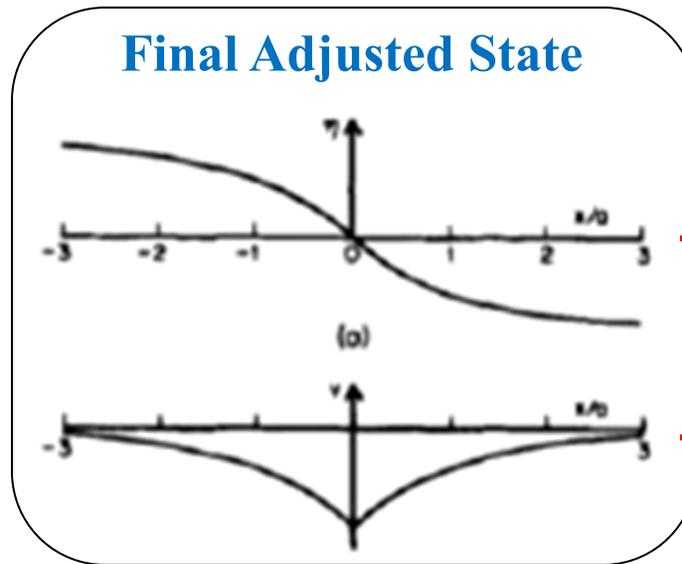
The n th baroclinic Rossby radius is:

$$L_{R,n} \equiv \frac{NH}{n\pi f_0}, \text{ where } N \text{ is the Brunt-Väisälä frequency, } H \text{ is the scale height, and } n = 1, 2, \dots$$

- The **gravity wave speed, and thus the Rossby radius,** increases proportionally with the depth of the disturbance.
- The gravity wave speed, and thus the Rossby radius, increases with stability by around a factor of two from steep lapse rates to isothermal conditions.



Final Adjusted State



Radius of deformation:

$$a = c/|f| = (gH)^{1/2}/|f|$$

$$\frac{\eta}{\eta_0} = \begin{cases} -1 + e^{-x/a} & \text{for } x > 0 \\ 1 - e^{x/a} & \text{for } x < 0, \end{cases}$$

$$v = -(g\eta_0/f a) \exp(-|x|/a).$$

- The steady equilibrium solution is *not one of rest, but is a geostrophic balance*.
- The equation determining this steady solution contains a length scale a , called the Rossby radius of deformation.
- The energy analysis indicates that *energy is hard to extract from a rotating fluid*. In the problem studied, there was an infinite amount of potential energy available for conversion into kinetic energy, but only a finite amount of this available energy was released. The reason was that a geostrophic equilibrium was established, and such an equilibrium retains potential energy.

Rossby Radius of Deformation

For Barotropic Flow

$$L_R \equiv \frac{(gD)^{1/2}}{f_0}$$

water depth

For Baroclinic Flow

Brunt-Vaisala frequency

$$L_R \equiv \frac{NH}{f_0}$$

equivalent depth

- In atmospheric dynamics and physical oceanography, the Rossby radius of deformation is the length scale at which *rotational effects* become as important as *buoyancy or gravity wave effects* in the evolution of the flow about some disturbance.
- “deformation”: It is the radius that the direction of the flow will be “deformed” by the Coriolis force from straight down the pressure gradient to be in parallel to the isobars.
- The size of the radius depends on the stratification (how density or potential temperature changes with height) and Coriolis parameter.
- The Rossby radius is considerably larger near the equator.



Rossby Number

$$R_0 \equiv U / (f_0 L)$$

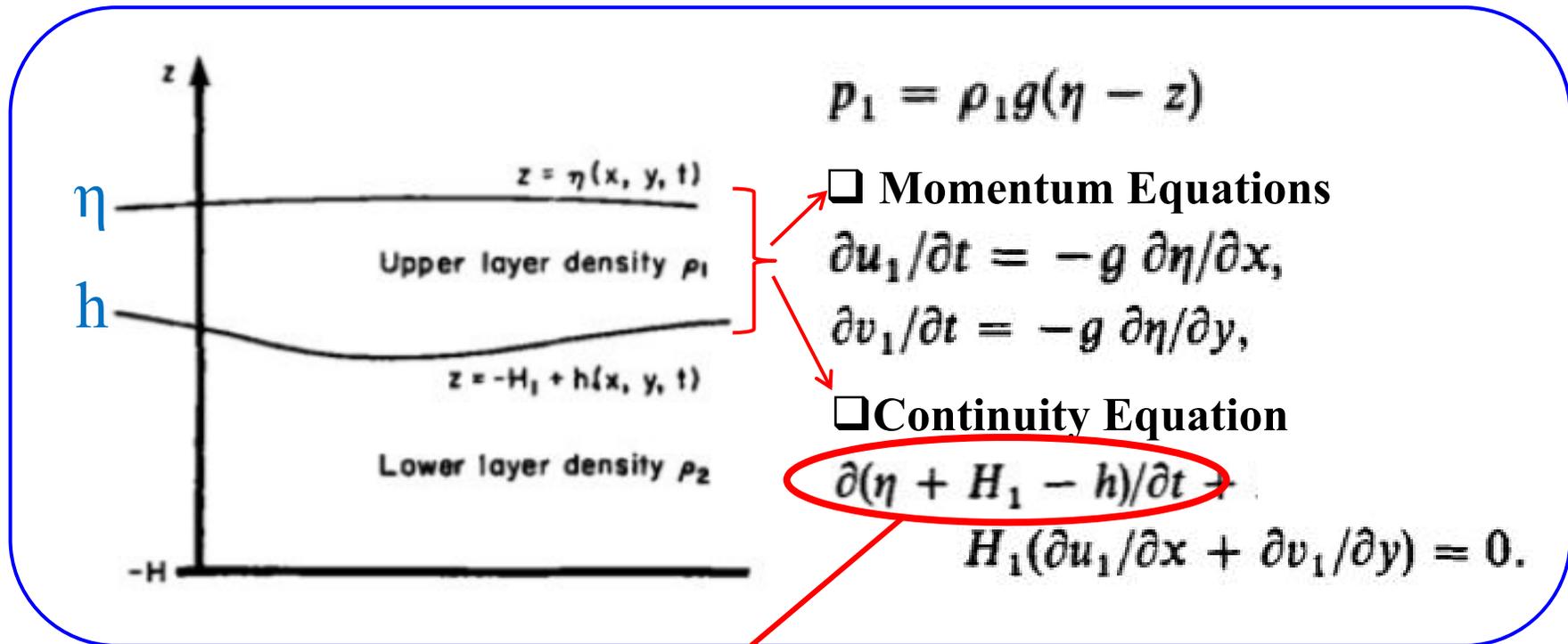
- Rossby number is a non-dimensional measure of the magnitude of the acceleration compared to the Coriolis force:

$$(U^2/L)/(f_0 U)$$

- The smaller the Rossby number, the better the geostrophic balance can be used.



Rigid Lid Approximation (for the upper layer)



- For baroclinic modes, surface displacements (η) are small compared to interface displacements (h).
- If there is a rigid lid at $z=0$, the identical pressure gradients would have been achieved.

$$-\partial h / \partial t + H_1(\partial u_1 / \partial x + \partial v_1 / \partial y) = 0.$$

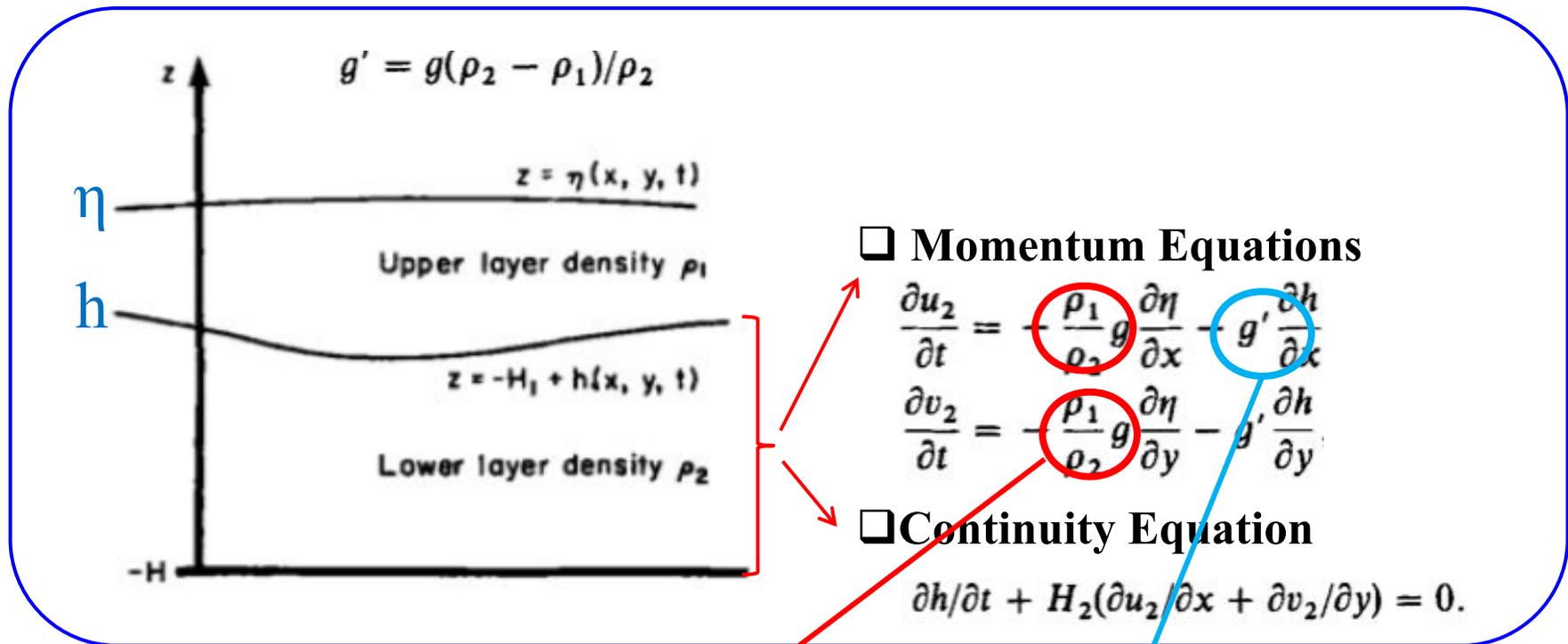


Purpose of Rigid Lid Approximation

- Rigid lid approximation: the upper surface was held fixed but could support pressure changes related to waves of lower speed and currents of interest.
- Ocean models used ed the "rigid lid" approximation to eliminate high-speed external gravity waves and allow a longer time step.
- As a result, ocean tides and other waves having the speed of tsunamis were filtered out.
- The **rigid lid approximation** was used in the 70's to filter out gravity wave dynamics in ocean models. Since then, ocean model have evolved to include a free-surface allowing fast-moving gravity wave physics.



Boussinesq Approximation (for the lower layer)



- Boussinesq approx: replace the ratio (ρ_1/ρ_2) by unity in the momentum equation.

$$\frac{\partial u_2}{\partial t} = -g \frac{\partial \eta}{\partial x} - g' \frac{\partial h}{\partial x},$$

$$\frac{\partial v_2}{\partial t} = -g \frac{\partial \eta}{\partial y} - g' \frac{\partial h}{\partial y}.$$

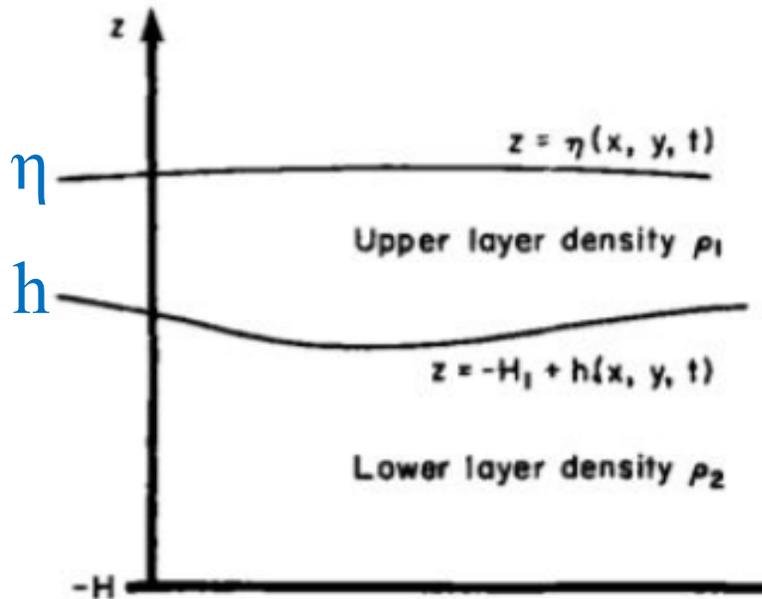
- We keep the density difference in this g' term, because it involves density difference $(\rho_1 - \rho_2)/\rho_1 * g$, which is related to the buoyancy force.

Purpose of Boussinesq Approximation

- This approximation states that density differences are sufficiently small to be neglected, except where they appear in terms multiplied by g , the acceleration due to gravity (i.e., buoyancy).
- In the Boussinesq approximation, which is appropriate for an almost-incompressible fluid, it is assumed that variations of density are small, so that in the inertial terms, and in the continuity equation, we may substitute ρ by ρ_0 , a constant. However, even weak density variations are important in buoyancy, and so we retain variations in ρ in the buoyancy term in the vertical equation of motion.
- *Sound waves are impossible/neglected when the Boussinesq approximation is used, because sound waves move via density variations.*
- Boussinesq approximation is for the problems that the variations of temperature as well as the variations of density are small. In these cases, the variations in volume expansion due to temperature gradients will also be small. For these cases, Boussinesq approximation can simplify the problems and save computational time.



After Using the Two Approximations



□ Upper layer

$$\partial u_1 / \partial t = -g \partial \eta / \partial x,$$

$$\partial v_1 / \partial t = -g \partial \eta / \partial y,$$

$$-\partial h / \partial t + H_1 (\partial u_1 / \partial x + \partial v_1 / \partial y) = 0.$$

□ Lower layer

$$\partial u_2 / \partial t = -g \partial \eta / \partial x - g' \partial h / \partial x,$$

$$\partial v_2 / \partial t = -g \partial \eta / \partial y - g' \partial h / \partial y.$$

$$\partial h / \partial t + H_2 (\partial u_2 / \partial x + \partial v_2 / \partial y) = 0.$$

- After the approximations, there is no η in the two continuity equations \rightarrow They can be combined to become one equation.
- The two momentum equations can also be combined into one single equation without η .
- At the end, the continuity and momentum equations for the upper and lower layers can be combined to solve for the dispersive relation for the baroclinic mode of the gravity wave.



Brunt–Väisälä Frequency (N)

- Consider a parcel of (water or gas) that has density of ρ_0 and the environment with a density that is a function of height: $\rho = \rho(z)$. If the parcel is displaced by a small vertical increment z' , it will subject to an extra gravitational force against its surroundings of:

$$\rho_0 \frac{\partial^2 z'}{\partial t^2} = -g(\rho_0 - \rho(z'))$$
$$\rho(z) - \rho_0 = \frac{\partial \rho(z)}{\partial z} z'$$

$$\rightarrow \frac{\partial^2 z'}{\partial t^2} = \frac{g}{\rho_0} \frac{\partial \rho(z)}{\partial z} z'$$

$$\rightarrow z' = z'_0 e^{\sqrt{-N^2}t}$$

where

$$N = \sqrt{-\frac{g}{\rho_0} \frac{\partial \rho(z)}{\partial z}}$$

for oceans

$$N \equiv \sqrt{\frac{g}{\theta} \frac{d\theta}{dz}},$$

for the atmosphere

- A fluid parcel in the presence of stable stratification ($N^2 > 0$) will oscillate vertically if perturbed vertically from its starting position.
- In atmospheric dynamics, oceanography, and geophysics, the Brunt-Vaisala frequency, or buoyancy frequency, is the angular frequency at which a vertically displaced parcel will oscillate within statically stable environment.
- The Brunt–Väisälä frequency relates to internal gravity waves and provides a useful description of atmospheric and oceanic stability.



Internal Gravity Waves in Atmosphere and Oceans

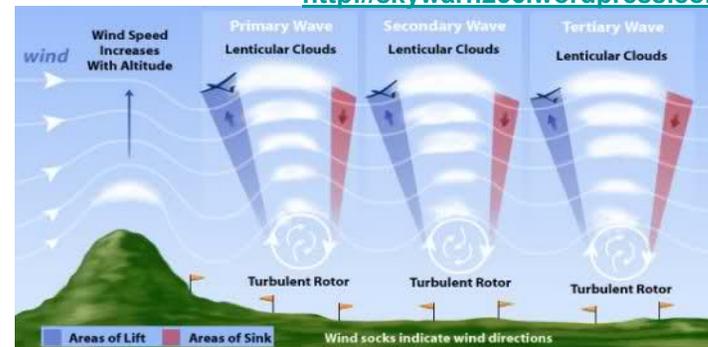
<http://skywarn256.wordpress.com>

In Oceans

$$N = \sqrt{-\frac{g}{\rho_0} \frac{\partial \rho(z)}{\partial z}}$$

In Atmosphere

$$N \equiv \sqrt{\frac{g}{\theta} \frac{d\theta}{dz}}$$



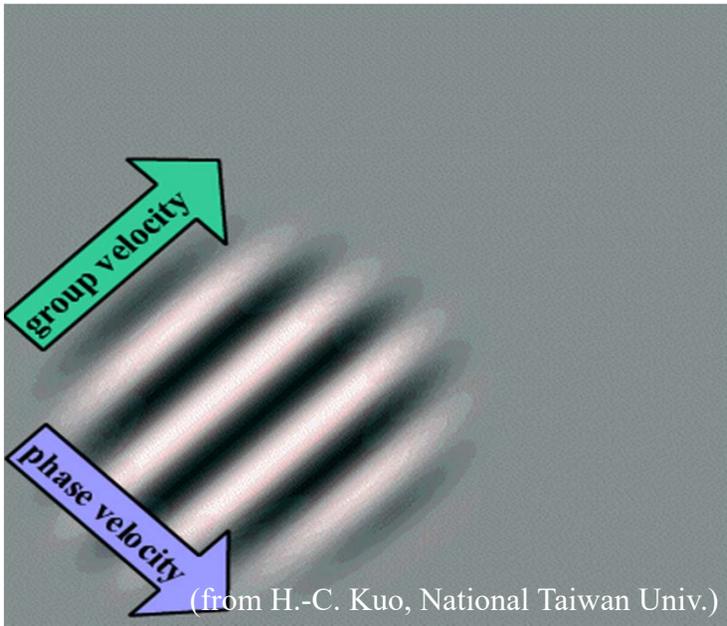
- ❑ Internal gravity waves can be found in both the *statically stable* ($d\theta/dz > 0$) atmosphere and the *stably stratified* ($-d\rho/dz > 0$) ocean.
- ❑ The buoyancy frequency for the internal gravity wave in the ocean is determined by the vertical density gradient, while it is determined by the vertical gradient of potential temperature in the atmosphere.
- ❑ In the troposphere, the typical value of N is 0.01 sec^{-1} , which correspond to a period of about 10 minutes.
- ❑ Although there are plenty of gravity waves in the atmosphere, most of them have small amplitudes in the troposphere and are not important, except that the gravity waves generated by flows over mountains. These mountain waves can have large amplitudes.
- ❑ Gravity waves become more important when they propagate into the upper atmosphere (particularly in the mesosphere) where their amplitudes got amplified due to low air density there.



Dispersion of Internal Gravity Waves

$$\hat{v} \equiv v - \overline{u}k = \pm Nk / \left(k^2 + m^2 \right)^{1/2} = \pm Nk / |\kappa|$$

mean flow
zonal wavenumber
vertical wavenumber
total wavenumber



\hat{v} is always smaller than N !!

Internal gravity waves can have any frequency between zero and a maximum value of N .

- Phase velocity:

$$c_x = \hat{v} / k \text{ and } , c_z = \hat{v} / m$$

- Group velocity:

$$c_{gx} = \frac{\partial v}{\partial k} = \overline{u} \pm \frac{Nm^2}{(k^2 + m^2)^{3/2}}$$

$$c_{gz} = \frac{\partial v}{\partial m} = \pm \frac{(-Nkm)}{(k^2 + m^2)^{3/2}}$$

- In the atmosphere, internal gravity waves generated in the troposphere by cumulus convection, by flow over topography, and by other processes may propagate upward many scale heights into the middle atmosphere.

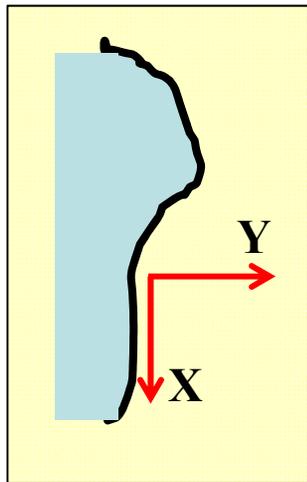
- The phase velocity and group velocity are perpendicular and that the vertical components of the phase and group velocities have opposite sign: if a wavepacket moves upward to the right, the crests move downward to the right.

Kelvin Waves

- ❑ The Kelvin wave is a large-scale wave motion of great practical importance in the Earth's atmosphere and ocean.
- ❑ The Kelvin wave is a special type of gravity wave that is affected by the Earth's rotation and trapped at the Equator or along lateral vertical boundaries such as coastlines or mountain ranges.
- ❑ The existence of the Kelvin wave relies on (a) gravity and stable stratification for sustaining a gravitational oscillation, (b) significant Coriolis acceleration, and (c) the presence of vertical boundaries or the equator.
- ❑ There are two basic types of Kelvin waves: boundary trapped and equatorially trapped. Each type of Kelvin wave may be further subdivided into surface and internal Kelvin waves.
- ❑ Atmospheric Kelvin waves play an important role in the adjustment of the tropical atmosphere to convective latent heat release, in the stratospheric quasi-biennial oscillation, and in the generation and maintenance of the Madden–Julian Oscillation.
- ❑ Oceanic Kelvin waves play a critical role in tidal motion, in the adjustment of the tropical ocean to wind stress forcing, and in generating and sustaining the El Niño Southern Oscillation.

(from Bin Wang 2002)

Kelvin Waves



Governing Equations	
$\frac{du}{dt} - fv$	$= -g \frac{\partial h}{\partial x}$;
$\frac{dv}{dt} + fu$	$= -g \frac{\partial h}{\partial y}$;
$\frac{dh}{dt} + D \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$	$= 0$.
A unique boundary condition	
$y = 0$	is $v = 0$



$\begin{pmatrix} u' \\ h' \end{pmatrix} = \text{Re} \left\{ \begin{pmatrix} U(y) \\ H(y) \end{pmatrix} \exp [ik(x - ct)] \right\}$
$H = \text{const} \times \exp \left(-\frac{f}{c} y \right)$
$-\frac{f}{g} U = -\frac{f}{c} H$
$c = \sqrt{gD}$

- A Kelvin wave is a type of low-frequency gravity wave in the ocean or atmosphere that balances the Earth's Coriolis force against a topographic boundary such as a coastline, or a waveguide such as the equator.
- Therefore, there are two types of Kelvin waves: coastal and equatorial.
- A feature of a Kelvin wave is that it is non-dispersive, i.e., the phase speed of the wave crests is equal to the group speed of the wave energy for all frequencies.



Costal Kelvin Waves

$$H = \text{const} \times \exp\left(-\frac{f}{c}y\right)$$

At the coast $y = 0$ is $v = 0$:

$$c = \sqrt{gD}$$

depth of the fluid

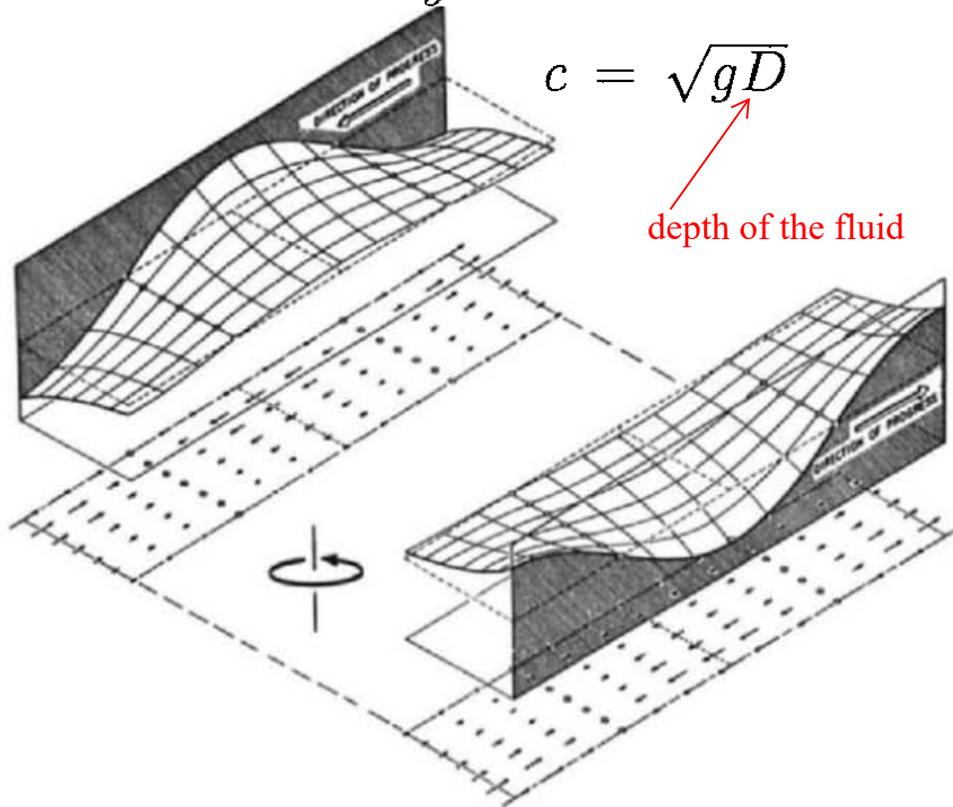
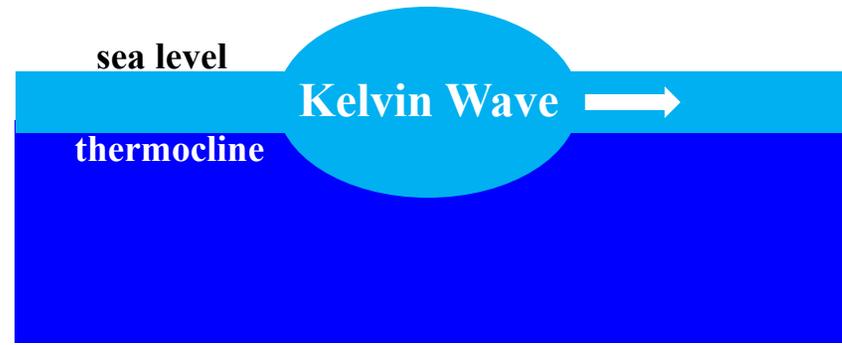


Fig. 10.3. Northern hemisphere Kelvin waves on opposite sides of a channel that is wide compared with the Rossby radius. In each vertical plane parallel to the coast, the currents (shown by arrows) are entirely within the plane and are exactly the same as those for a long gravity wave in a nonrotating channel. However, the surface elevation varies exponentially with distance from the coast in order to give a geostrophic balance. This means Kelvin waves move with the coast on their right in the northern hemisphere and on their left in the southern hemisphere. [From Mortimer (1977).]

- Coastal Kelvin waves always propagate with the shoreline on the right in the northern hemisphere and on the left in the southern hemisphere.
- In each vertical plane to the coast, the currents (shown by arrows) are entirely within the plane and are exactly the same as those for a long gravity wave in a non-rotating channel.
- However, the surface elevation varies exponentially with distance from the coast in order to give a geostrophic balance.



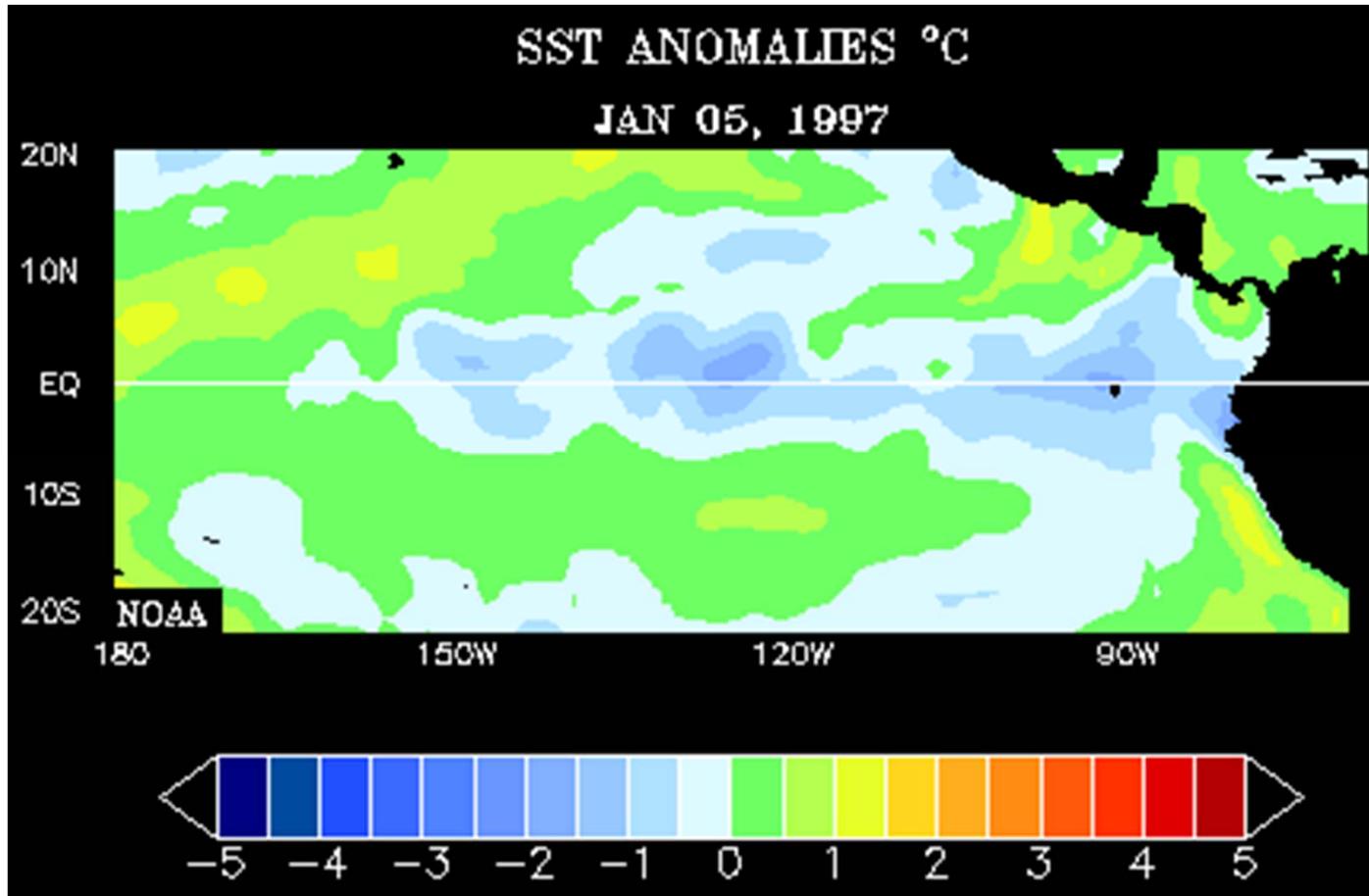
Equatorial Kelvin Waves



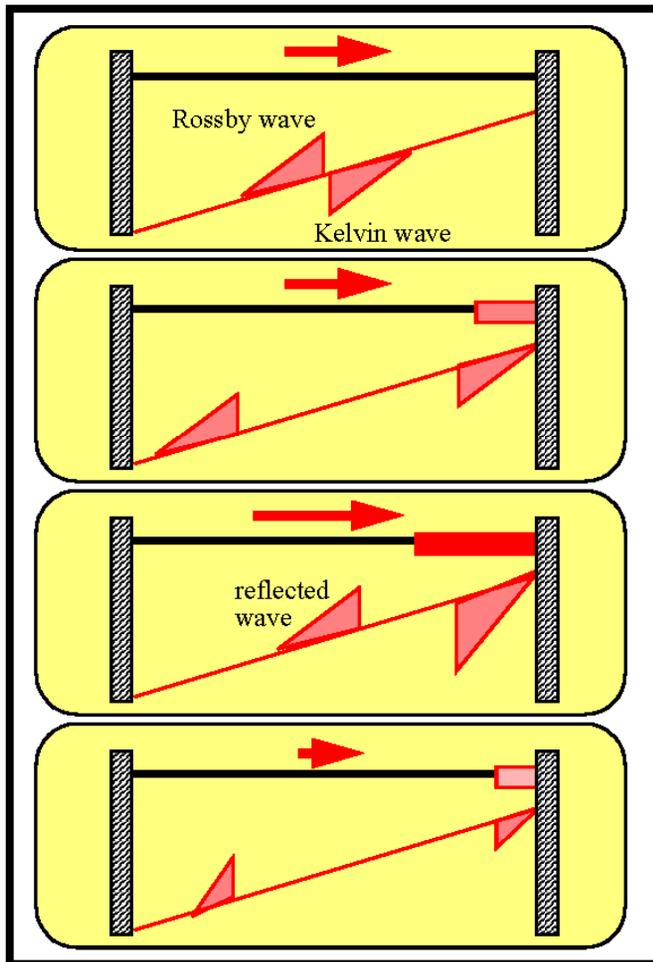
- The equator acts analogously to a topographic boundary for both the Northern and Southern Hemispheres, which make the equatorial Kelvin wave to behaves very similar to the coastally-trapped Kelvin wave.
- Surface equatorial Kelvin waves travel very fast, at about 200 m per second. Kelvin waves in the thermocline are however much slower, typically between 0.5 and 3.0 m per second.
- They may be detectable at the surface, as sea-level is slightly raised above regions where the thermocline is depressed and slightly depressed above regions where the thermocline is raised.
- The amplitude of the Kelvin wave is several tens of meters along the thermocline, and the length of the wave is thousands of kilometres.
- Equatorial Kelvin waves can only travel eastwards.



1997-98 El Nino

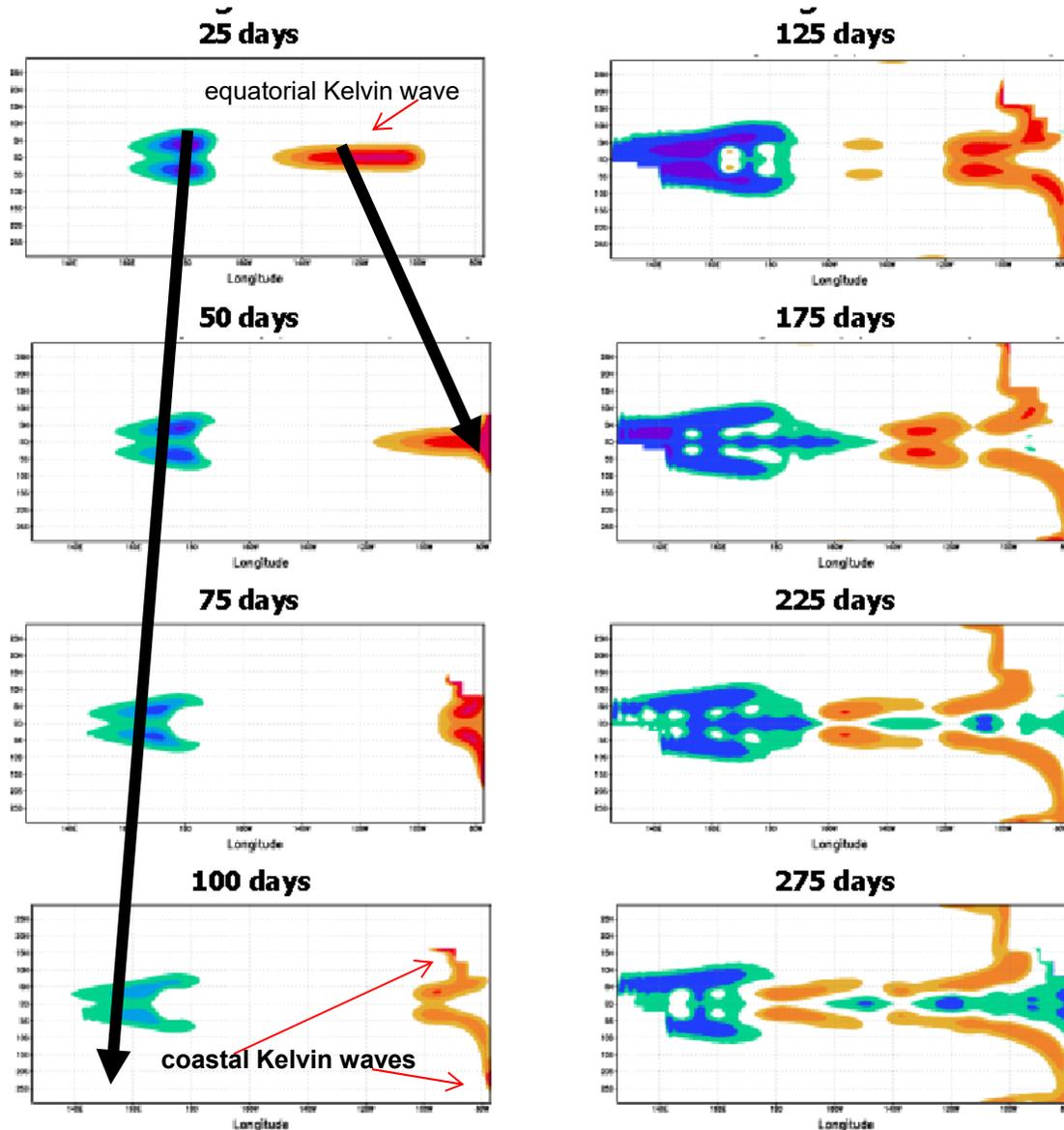


Delayed Oscillator Theory



- ❑ **Wind forcing at the central Pacific:** produces a downwelling Kelvin wave propagating eastward and an upwelling Rossby wave propagating westward.
- ❑ **wave propagation:** the fast Kelvin wave causes SST warming at the eastern basin, while the slow Rossby wave is reflected at the western boundary.
- ❑ **wave reflection:** the Rossby wave is reflected as an upwelling Kelvin wave and propagates back to the eastern basin to reverse the phase of the ENSO cycle.
- ❑ **ENSO period:** is determined by the propagation time of the waves.

Wave Propagation and Reflection

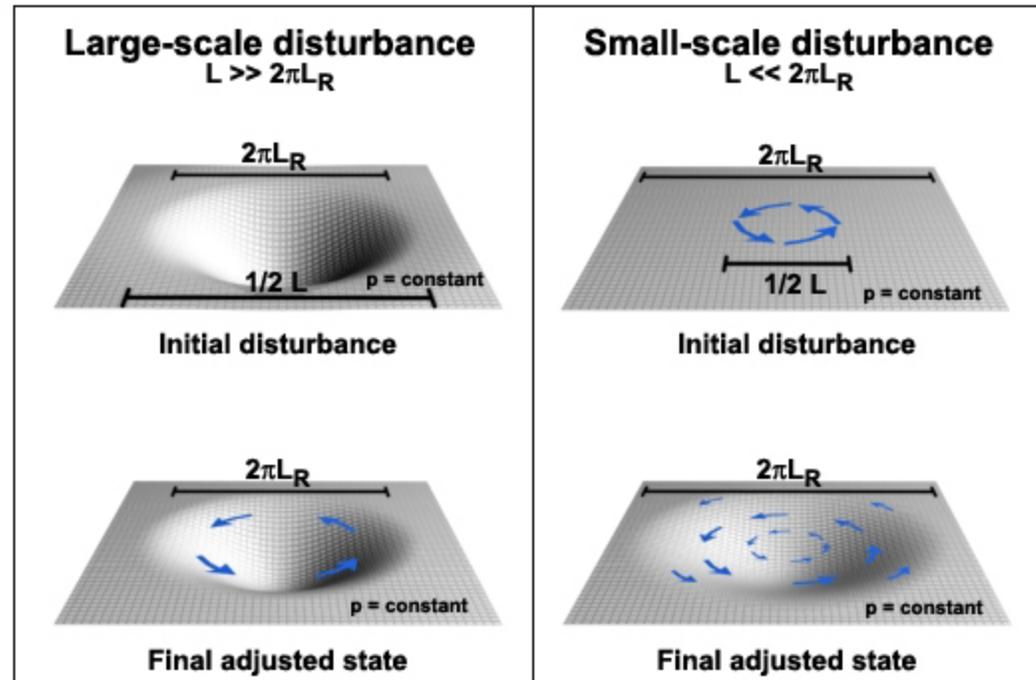


(Figures from IRI)

- ❑ It takes Kelvin wave (phase speed = 2.9 m/s for the first baroclinic mode) about 70 days to cross the Pacific basin (17,760km).
- ❑ It takes Rossby wave about 200 days (phase speed = 0.93 m/s) to cross the Pacific basin.



Lecture 8: Adjustment in a Rotating System



- Geostrophic Adjustment Process
- Rossby Radius of Deformation



Geostrophic Adjustments

- The atmosphere is nearly always close to geostrophic and hydrostatic balance.
- If this balance is disturbed through such processes as heating or cooling, the atmosphere adjusts itself to get back into balance. This process is called *geostrophic adjustment*.
- A key feature in the geostrophic adjustment process is that pressure and velocity fields have to adjust to each other in order to reach a geostrophic balance. When the balance is achieved, the flow at any level is along the isobars.
- We can study the geostrophic adjustment by studying the adjustment in a barotropic fluid using the shallow-water equations.
- The results can be extended to a baroclinic fluid by using the concept of equivalent depth.



Geostrophic Adjustment Problem

shallow water model

$$\begin{cases} \frac{\partial u'}{\partial t} - f_0 v' = -g \frac{\partial h'}{\partial x} \\ \frac{\partial v'}{\partial t} + f_0 u' = -g \frac{\partial h'}{\partial y} \\ \frac{\partial h'}{\partial t} + H \left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} \right) = 0 \end{cases}$$

$$\frac{\partial^2 h'}{\partial t^2} - c^2 \left(\frac{\partial^2 h'}{\partial x^2} + \frac{\partial^2 h'}{\partial y^2} \right) + f_0 H \zeta' = 0$$

$$\frac{\partial \zeta'}{\partial t} + f_0 \left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} \right) = 0$$

$$\frac{\partial \zeta'}{\partial t} - \frac{f_0}{H} \frac{\partial h'}{\partial t} = 0$$

$$Q'(x, y, t) = \zeta' / f_0 - h' / H = \text{Const.}$$



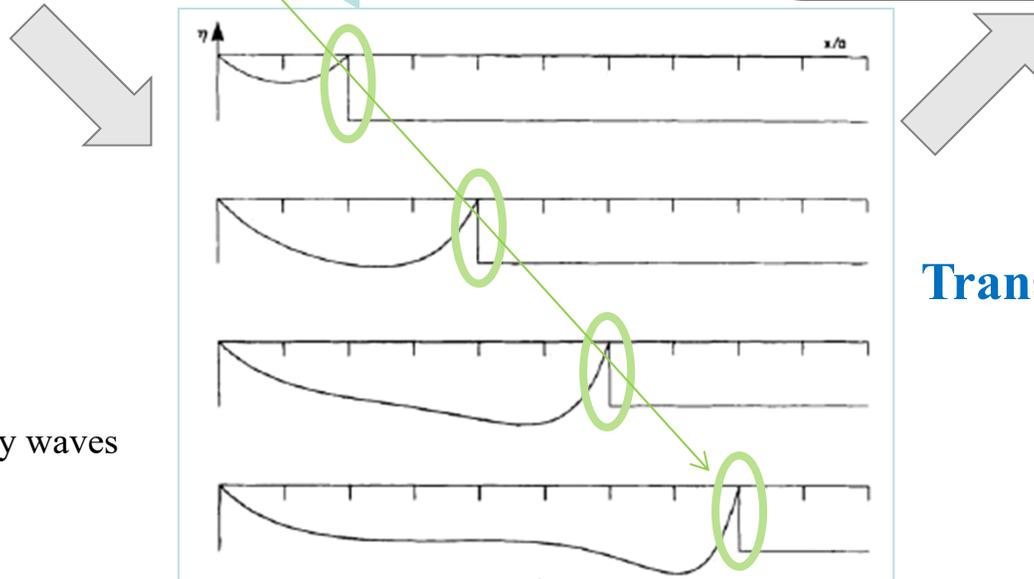
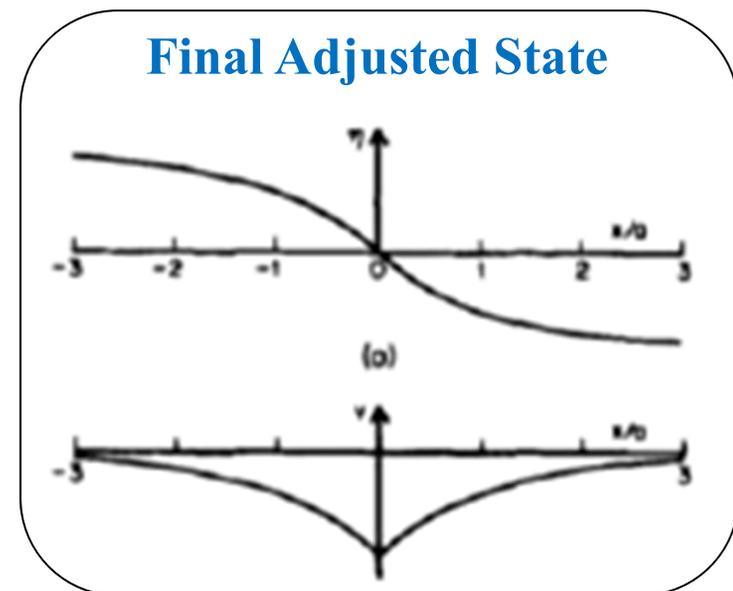
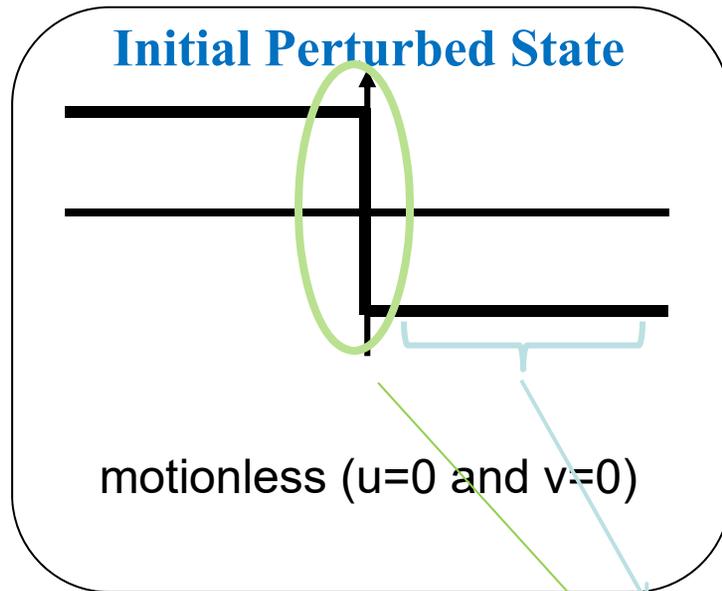
If we know the distribution of perturbation potential vorticity (Q') at the initial time, we know for all time:

$$Q'(x, y, t) = Q'(x, y, 0)$$

And the **final adjusted state** can be determined without solving the time-dependent problem.



An Example of Geostrophic Adjustment

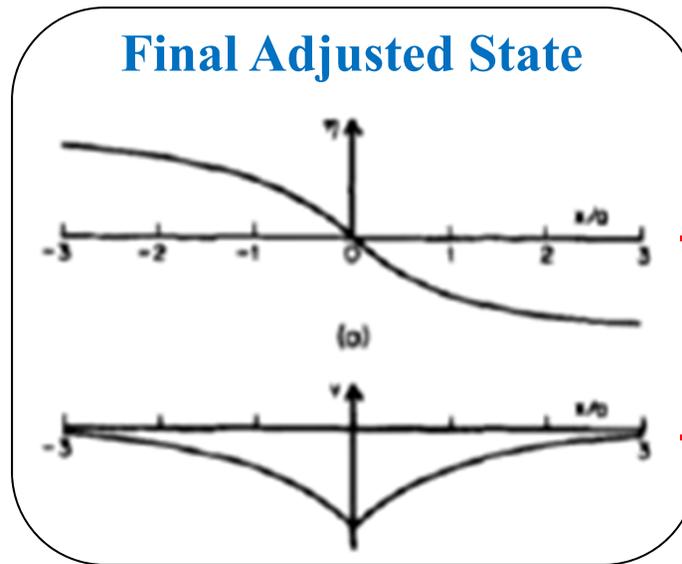


Inertial-gravity waves

Transient



Final Adjusted State



Radius of deformation:

$$a = c/|f| = (gH)^{1/2}/|f|$$

$$\frac{\eta}{\eta_0} = \begin{cases} -1 + e^{-x/a} & \text{for } x > 0 \\ 1 - e^{x/a} & \text{for } x < 0, \end{cases}$$

$$v = -(g\eta_0/f a) \exp(-|x|/a).$$

- The steady equilibrium solution is *not one of rest, but is a geostrophic balance*.
- The equation determining this steady solution contains a length scale a , called the Rossby radius of deformation.
- The energy analysis indicates that *energy is hard to extract from a rotating fluid*. In the problem studied, there was an infinite amount of potential energy available for conversion into kinetic energy, but only a finite amount of this available energy was released. The reason was that a geostrophic equilibrium was established, and such an equilibrium retains potential energy.

Rossby Radius of Deformation

For Barotropic Flow

$$L_R \equiv \frac{(gD)^{1/2}}{f_0}$$

water depth

For Baroclinic Flow

Brunt-Vaisala frequency

$$L_R \equiv \frac{NH}{f_0}$$

equivalent depth

- In atmospheric dynamics and physical oceanography, the Rossby radius of deformation is the length scale at which *rotational effects* become as important as *buoyancy or gravity wave effects* in the evolution of the flow about some disturbance.
- “deformation”: It is the radius that the direction of the flow will be “deformed” by the Coriolis force from straight down the pressure gradient to be in parallel to the isobars.
- The size of the radius depends on the stratification (how density or potential temperature changes with height) and Coriolis parameter.
- The Rossby radius is considerably larger near the equator.



Rossby Radius and the Equilibrium State

Mass and Velocity

$$-\frac{\zeta}{f} : \frac{\eta}{H} = \kappa_H^2 a^2 : 1$$

wave number \rightarrow deformation radius

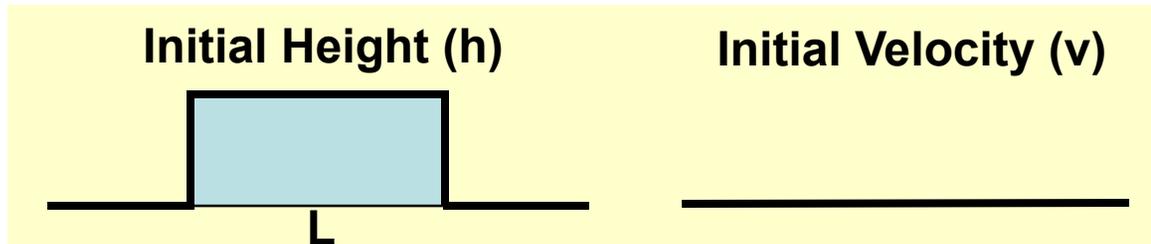
Energy Partition

$$\text{K.E.} : \text{P.E.} = \kappa_H^2 a^2 : 1,$$

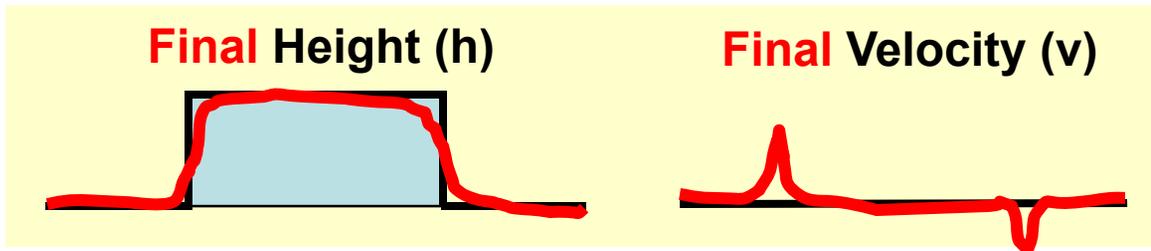
- For large scales ($K_H a \ll 1$), the potential vorticity perturbation is mainly associated with perturbations in the mass field, and that the energy changes are in the potential and internal forms.
- For small scales ($K_H a \gg 1$) potential vorticity perturbations are associated with the velocity field, and the energy perturbation is mainly kinetic.
- At large scales ($K_H^{-1} \gg a$; or $K_H a \ll 1$), it is the mass field that is determined by the initial potential vorticity, and the velocity field is merely that which is in geostrophic equilibrium with the mass field. It is said, therefore, that the large-scale velocity field adjusts to be in equilibrium with the large scale mass field.
- At small scales ($K_H^{-1} \ll a$) *it is the velocity field* that is determined by the initial potential vorticity, and the mass field is merely that which is in geostrophic equilibrium with the velocity field. In this case it can be said that the mass field adjusts to be in equilibrium with the velocity field.



Examples

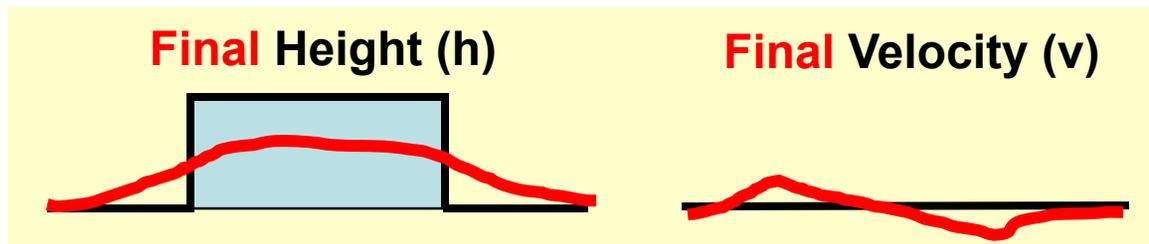


- If the Rossby radius of deformation is very small (i.e., $L_R \ll L$),



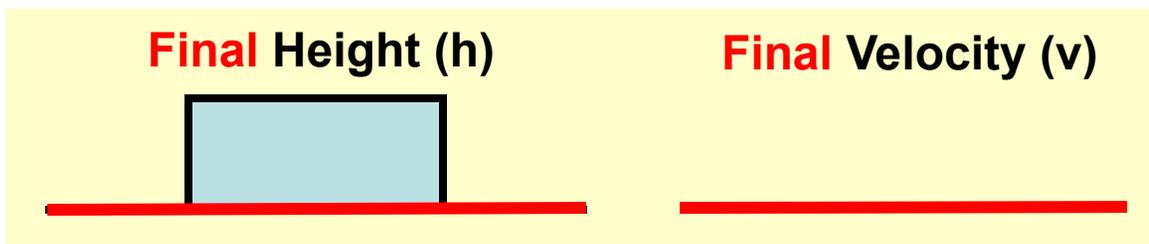
velocity
adjusted to
mass

- If the Rossby radius of deformation is comparable with L (i.e., $L_R = L$),



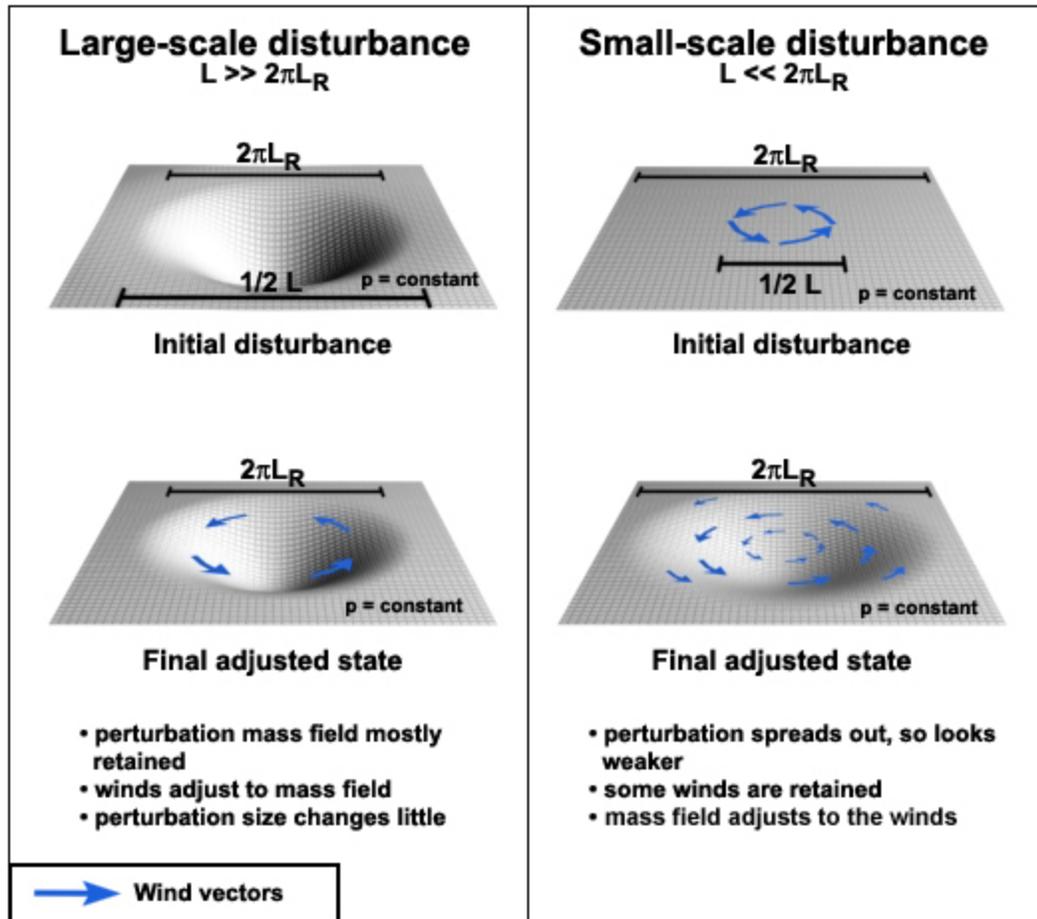
velocity and
mass both
adjusted

- If the Rossby radius of deformation is very large (i.e., $L_R \gg L$),



mass
adjusted to
velocity

Rossby Radius and the Equilibrium State



The COMET Program

- If the size of the disturbance is much larger than the Rossby radius of deformation, then the velocity field adjusts to the initial mass (height) field.
- If the size of the disturbance is much smaller than the Rossby radius of deformation, then the mass field adjusts to the initial velocity field.
- If the size of the disturbance is close to the Rossby radius of deformation, then both the velocity and mass fields undergo mutual adjustment.

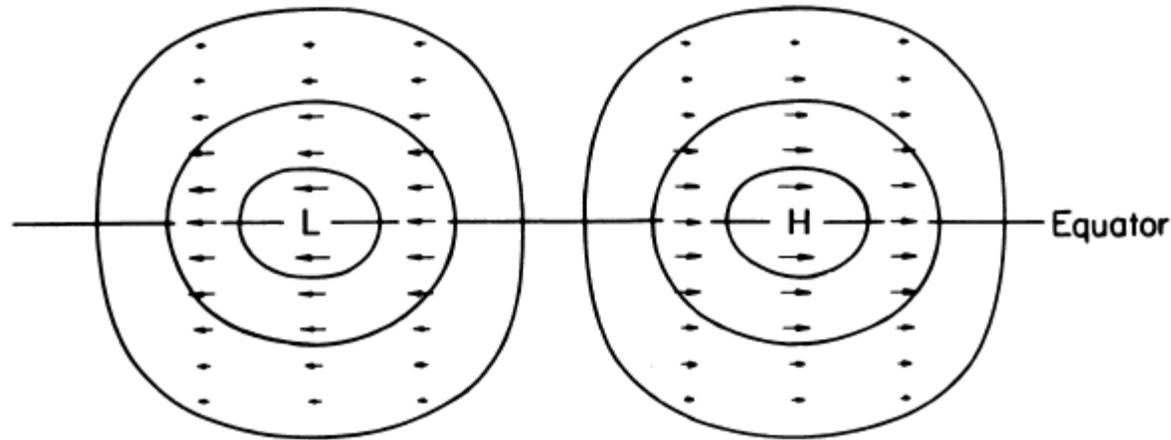


What does the Geostrophic Adjustment Tell Us?

- An important feature of the response of a rotating fluid to gravity is that it does not adjust to a state of rest, but rather to a geostrophic equilibrium.
- The Rossby adjustment problem explains why the atmosphere and ocean are nearly always close to geostrophic equilibrium, for if any force tries to upset such an equilibrium, the gravitational restoring force acts quickly to restore a near-geostrophic equilibrium.
- For deep water in the ocean, where H is 4 or 5 km, c is about 200 m/s and therefore the Rossby radius $a = c/f \sim 2000$ km.
- Near the continental shelves, such as for the North Sea where $H=40$ m, the Rossby radius $a = c/f \sim 200$ km. Since the North Sea has larger dimensions than this, rotation has a strong effect on transient motions such as tides and surges in that ocean region.



Lecture 9: Tropical Dynamics



- Equatorial Beta Plane
- Equatorial Wave Theory
- Equatorial Kelvin Wave
- Adjustment under Gravity near the Eq.
- Gill Type Response

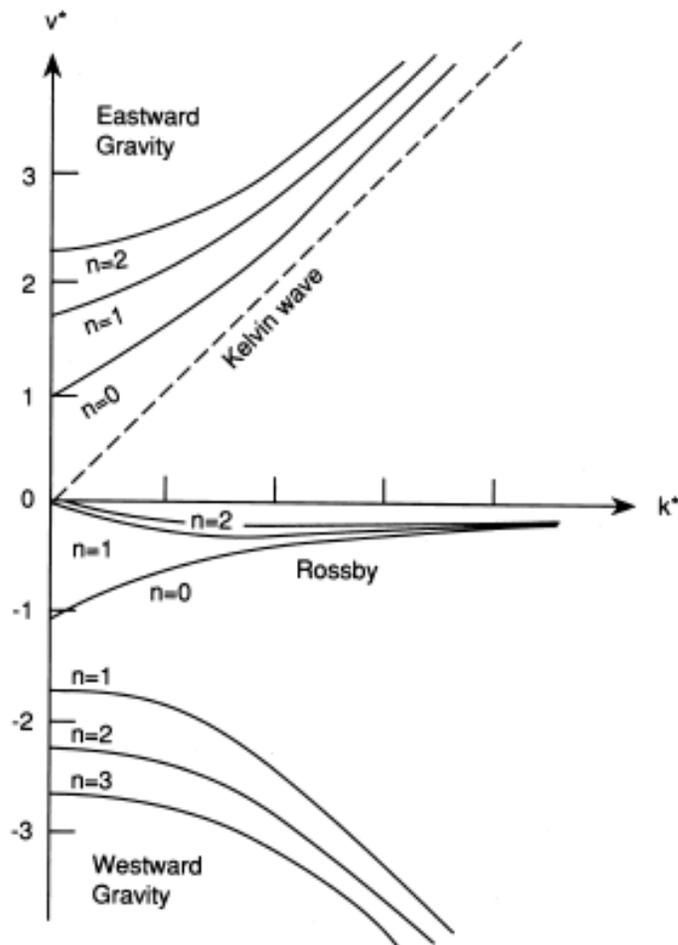


Overview

- In the **Mid-latitudes**, the **primary energy source** for synoptic-scale disturbances is the zonal available potential energy associated with the **latitudinal temperature gradient**; and latent heat release and radiative heating are usually secondary contributors.
- In the **tropics**, however, the storage of available potential energy is small due to the very small temperature gradients in the tropical atmosphere. **Latent heat release appears to be the primary energy source**.
- The dynamics of tropical circulations is very complicated, and there is no simple theoretical framework, analogous to quasi-geostrophic theory for the mid-latitude dynamics, that can be used to provide an overall understanding of large-scale tropical motions → **no simple pressure-wind relationship**.



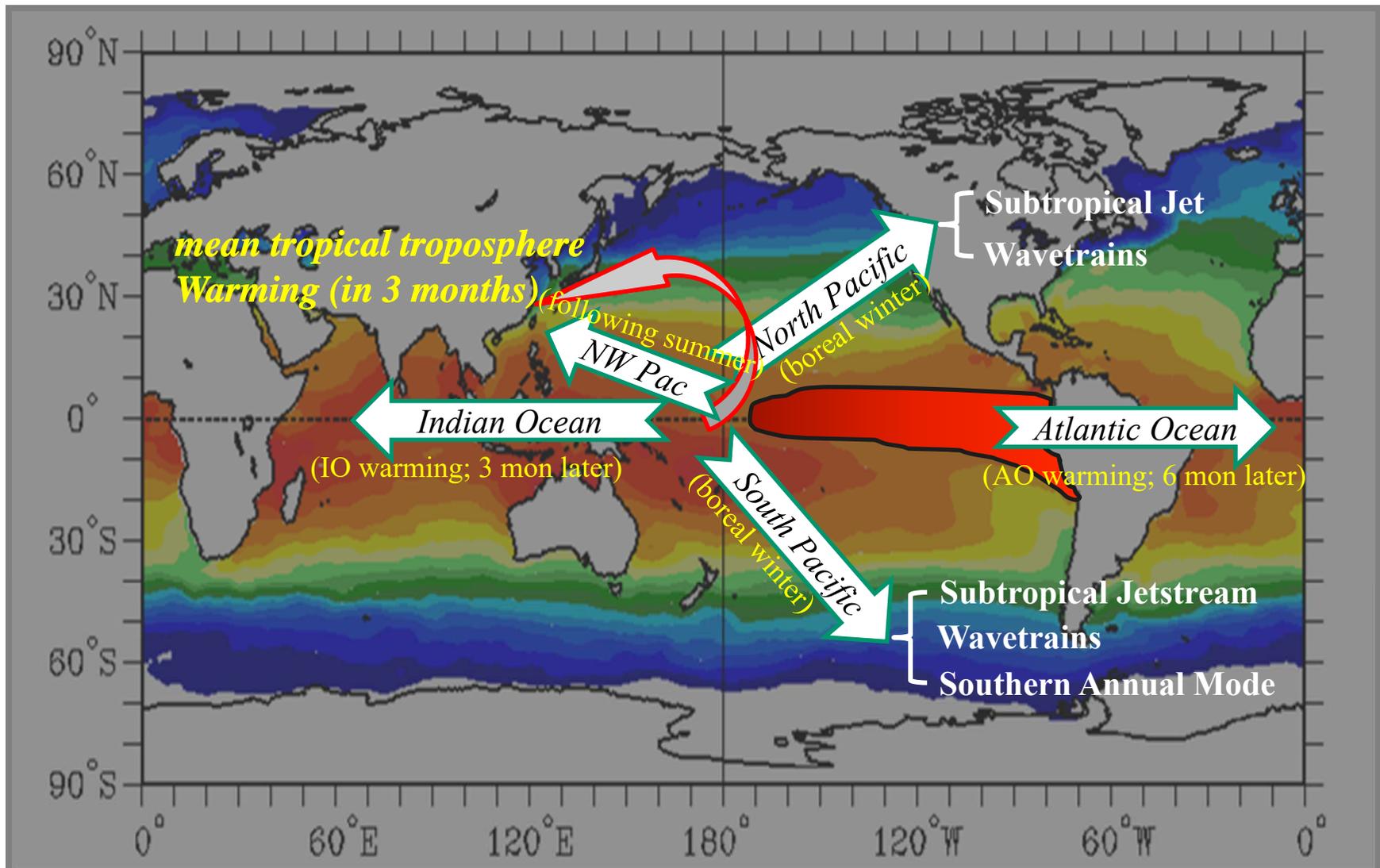
Equatorial Waves



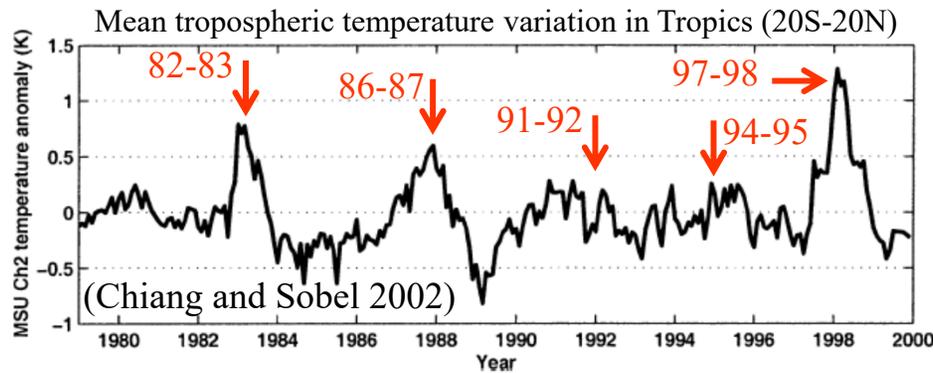
- **Equatorial waves** are an important class of **eastward and westward propagating** disturbances in the atmosphere and in the ocean that are trapped about the equator (i.e., they **decay away from the equatorial region**).
- **Diabatic heating** by organized tropical convection can **excite atmospheric equatorial waves**, whereas **wind stresses** can **excite oceanic equatorial waves**.
- Atmospheric equatorial wave propagation can cause the effects of convective storms to be communicated over large longitudinal distances, thus producing remote responses to localized heat sources.



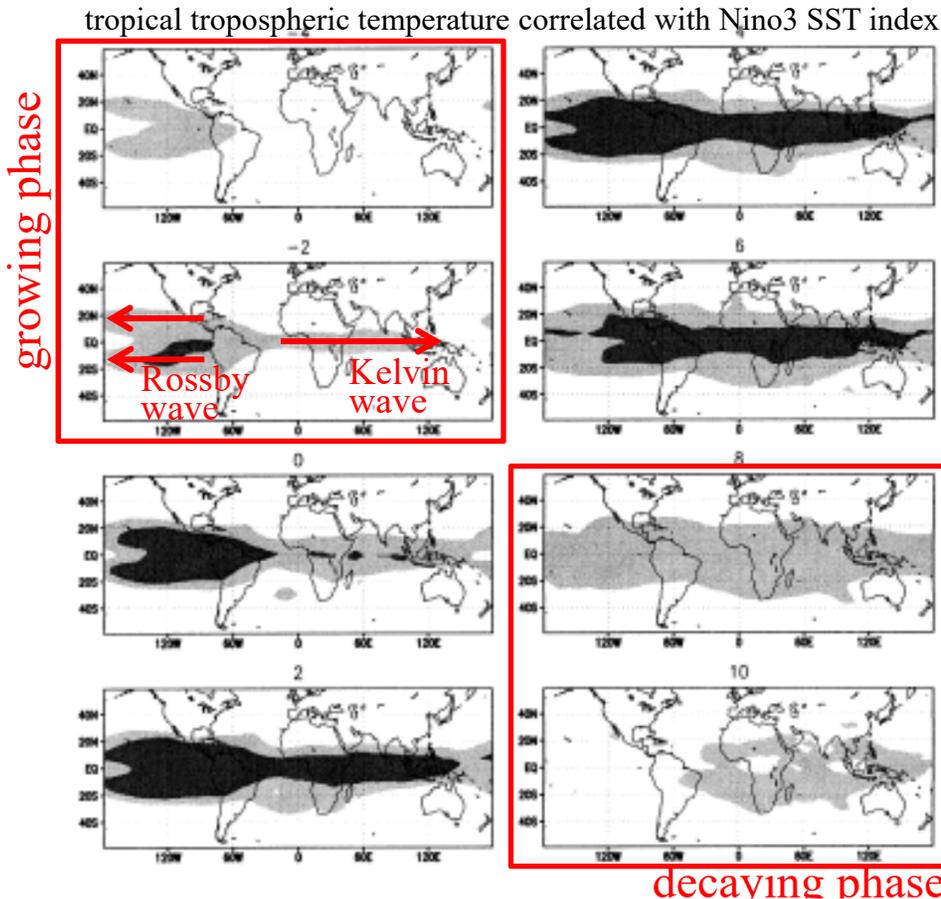
El Nino Teleconnection



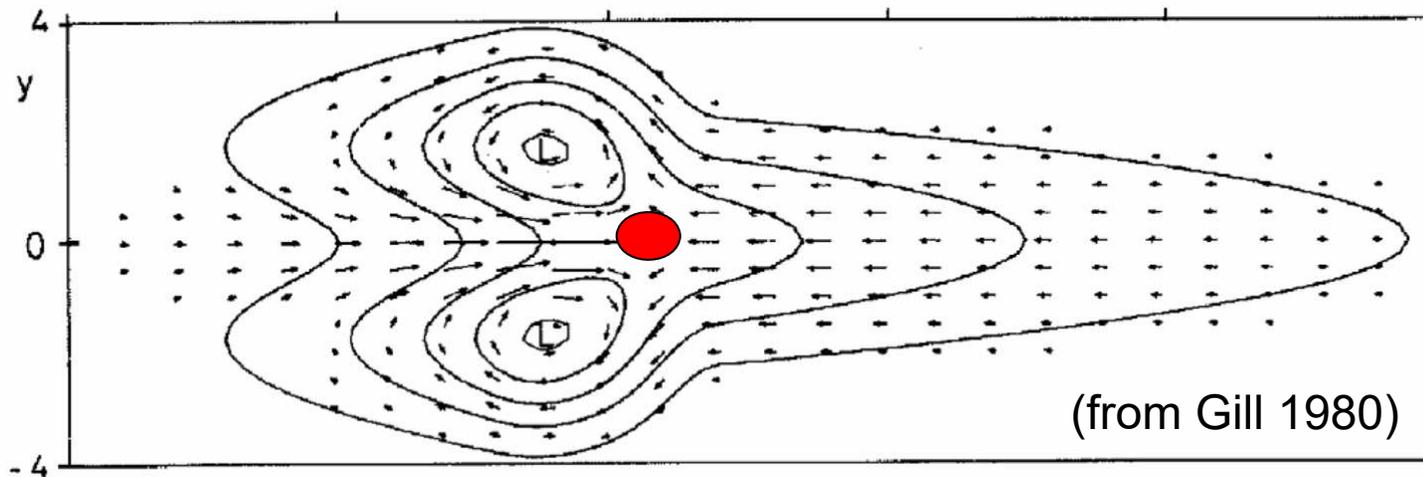
Tropical Tropospheric Temperature Response to ENSO



- ❑ During El Nino events, tropospheric temperature warms uniformly (0.5° - 1°C) over the global tropical strip.
- ❑ The tropical free atmosphere cannot maintain horizontal pressure gradients, and temperature anomalies become uniformly distributed over the global Tropics on timescales of a month or two.
- ❑ The Kelvin and Rossby wave responses to the El Nino forcing spread the tropospheric warming from eastern-to-central Pacific to other ocean basins.
- ❑ The lagged response time in other basins may be affected by air-sea coupling in the remote basins.



Gill's Response to Symmetric Heating



- This response consists of a eastward-propagating Kelvin wave to the east of the symmetric heating and a westward-propagating Rossby wave of $n=1$ to the west.
- The Kelvin wave produces low-level easterlies to the east of the heating, while the Rossby wave produces low-level westerlies to the west.
- The easterlies are trapped to the equator due to the property of the Kelvin wave.
- The $n=1$ Rossby wave consists of two cyclones symmetric and straddling the equator.
- The meridional scale of this response is controlled by the equatorial Rossby radius, which is related to the β -effect and the stability and is typically of the order of 1000km.

Equatorial β -Plane Approximation

- *f*-plane approximation: On a rotating sphere such as the earth, *f* varies with the sine of latitude; in the so-called *f*-plane approximation, this variation is ignored, and a value of *f* appropriate for a particular latitude is used throughout the domain.
- *β* -plane approximation: *f* is set to vary linearly in space.
- The advantage of the beta plane approximation over more accurate formulations is that it does not contribute nonlinear terms to the dynamical equations; such terms make the equations harder to solve.
- Equatorial β -plane approximation:

$$\left\{ \begin{array}{l} \cos\varphi \approx 1, \\ \sin\varphi \approx y/a. \end{array} \right.$$

$$f \approx \beta y$$

$$\text{and } \beta = 2\Omega/r = 2.3 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1}$$

$$f = f_0 + \beta y_s = 2\Omega \sin\varphi + \frac{2\Omega \cos\varphi}{a} y_s$$

Very small near equator

therefore



Shallow-Water Equation on an Equatorial β -Plane

Linearized shallow-water equations

$$\begin{aligned} \partial u' / \partial t - \beta y v' &= -\partial \Phi' / \partial x \\ \partial v' / \partial t + \beta y u' &= -\partial \Phi' / \partial y \\ \partial \Phi' / \partial t + g h_e (\partial u' / \partial x + \partial v' / \partial y) &= 0 \end{aligned}$$

Assume wave-form solutions

$$\begin{pmatrix} u' \\ v' \\ \Phi' \end{pmatrix} = \begin{pmatrix} \hat{u}(y) \\ \hat{v}(y) \\ \hat{\Phi}(y) \end{pmatrix} \exp[i(kx - vt)]$$

$$\begin{aligned} -iv\hat{u} - \beta y\hat{v} &= -ik\hat{\Phi} \\ -iv\hat{v} + \beta y\hat{u} &= -\partial\hat{\Phi}/\partial y \\ -iv\hat{\Phi} + g h_e (ik\hat{u} + \partial\hat{v}/\partial y) &= 0 \end{aligned}$$

Only if this constant equal to an odd integer that the boundary condition ($v=0$ at $y=\infty$) can be satisfied.

$$\frac{\partial^2 \hat{v}}{\partial y^2} + \left[\left(\frac{v^2}{g h_e} - k^2 - \frac{k}{v} \beta \right) - \frac{\beta^2 y^2}{g h_e} \right] \hat{v} = 0$$

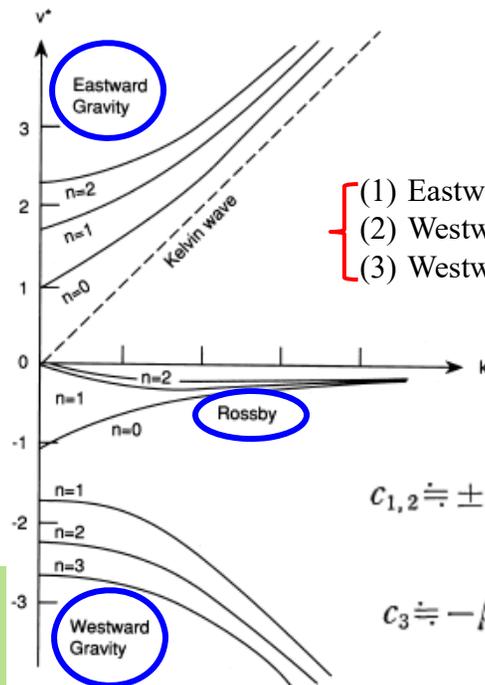
The index n corresponds to the number of nodes in the meridional velocity profile in the domain $|y| < \infty$.

Dispersion relationship for equatorial waves

$$\frac{\sqrt{g h_e}}{\beta} \left(-\frac{k}{v} \beta - k^2 + \frac{v^2}{g h_e} \right) = 2n + 1; \quad n = 0, 1, 2, \dots$$

$$\frac{\sqrt{g h_e}}{\beta} \left(-\frac{k}{v} \beta - k^2 + \frac{v^2}{g h_e} \right) = 2n + 1; \quad n = 0, 1, 2, \dots$$

This **cubic** dispersion equation permit **three groups** of equatorially trapped waves:



- (1) Eastward-moving gravity waves
- (2) Westward-moving gravity waves
- (3) Westward-moving Rossby waves

$$c_{1,2} \doteq \pm c_g \sqrt{1 + \frac{1}{k^2} \frac{\beta}{c_g} (2n+1)}$$

$$c_3 \doteq -\beta \left[k^2 + \frac{\beta}{c_g} (2n+1) \right]$$

Only these waves that satisfy the condition that the wave amplitudes decay far from the equator (where the beta-plane approximation becomes invalid.)



Equatorial Waveguide

Linearized shallow-water equations

$$\begin{aligned} \partial u' / \partial t - \beta y v' &= -\partial \Phi' / \partial x \\ \partial v' / \partial t + \beta y u' &= -\partial \Phi' / \partial y \\ \partial \Phi' / \partial t + g h_e (\partial u' / \partial x + \partial v' / \partial y) &= 0 \end{aligned}$$

Assume wave-form solutions

$$\begin{pmatrix} u' \\ v' \\ \Phi' \end{pmatrix} = \begin{pmatrix} \hat{u}(y) \\ \hat{v}(y) \\ \hat{\Phi}(y) \end{pmatrix} \exp[i(kx - \nu t)]$$

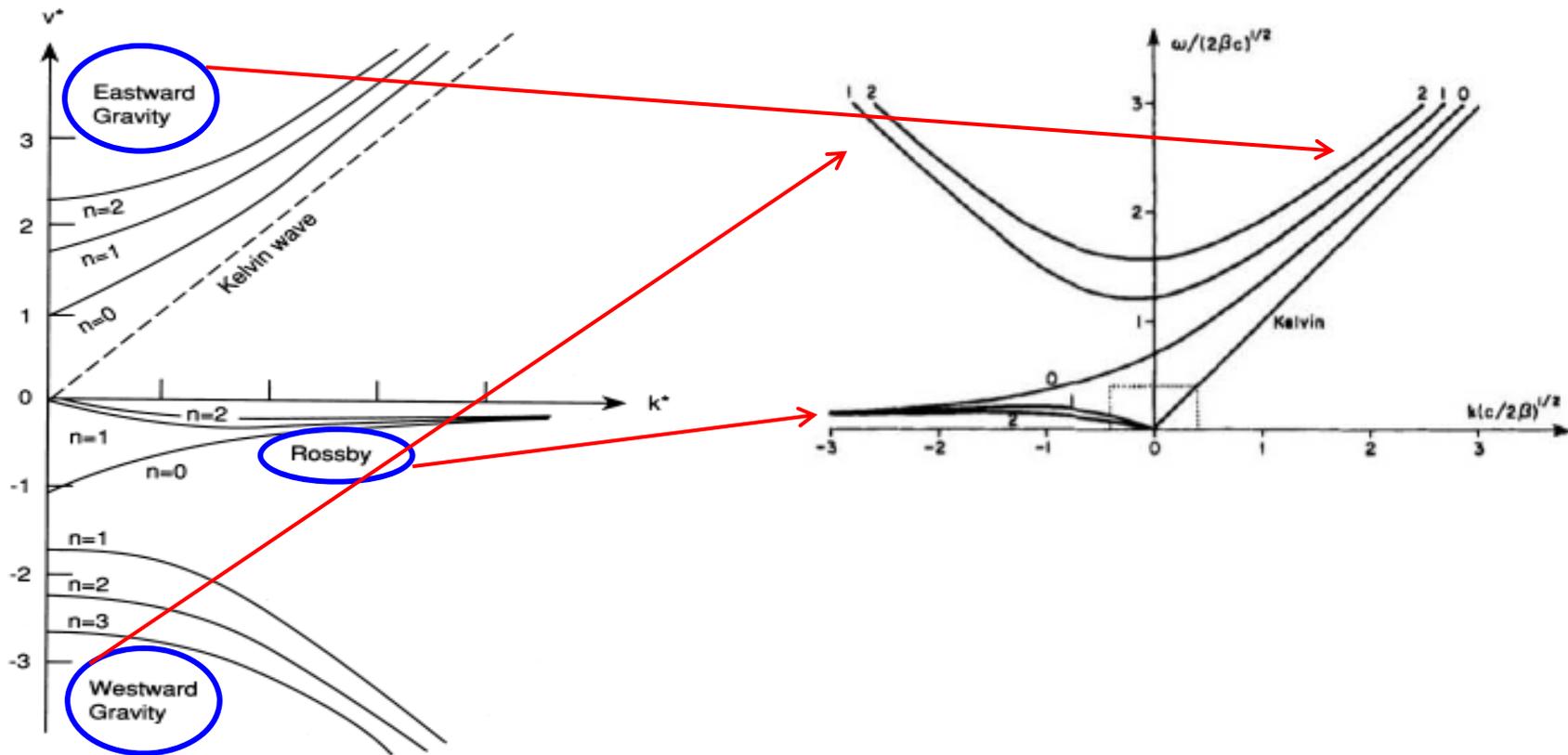
$$\begin{aligned} -i\nu \hat{u} - \beta y \hat{v} &= -ik \hat{\Phi} \\ -i\nu \hat{v} + \beta y \hat{u} &= -\partial \hat{\Phi} / \partial y \\ -i\nu \hat{\Phi} + g h_e (ik \hat{u} + \partial \hat{v} / \partial y) &= 0 \end{aligned}$$

$$\frac{\partial^2 \hat{v}}{\partial y^2} + \left[\left(\frac{\nu^2}{g h_e} - k^2 - \frac{k}{\nu} \beta \right) - \frac{\beta^2 y^2}{g h_e} \right] \hat{v} = 0$$

- Wave-like solutions can result from this equation if this coefficient has a positive value.
- When y increases (i.e., away from the equator), this coefficient becomes negative (due to the $\beta^2 y^2$ term) and the wave-like solution becomes an exponential solution (i.e., the wave solution disappears).
- Equatorial waves are trapped in the tropics.
- This waveguide effect is due entirely to the variation of Coriolis parameter with latitude.

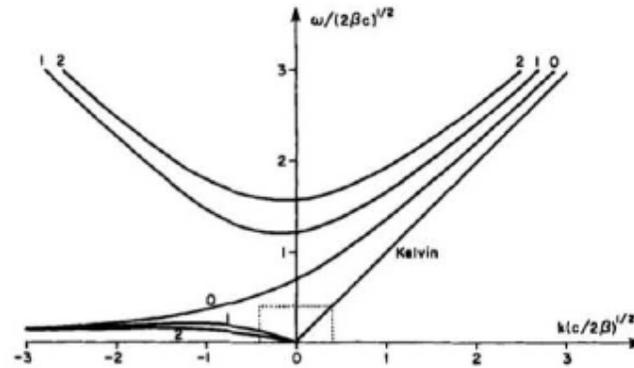


Three Groups of Equatorial Waves



Equatorial Waves with $n=1, 2, \dots$

(Eastward/Westward Gravity Waves; Rossby Waves)

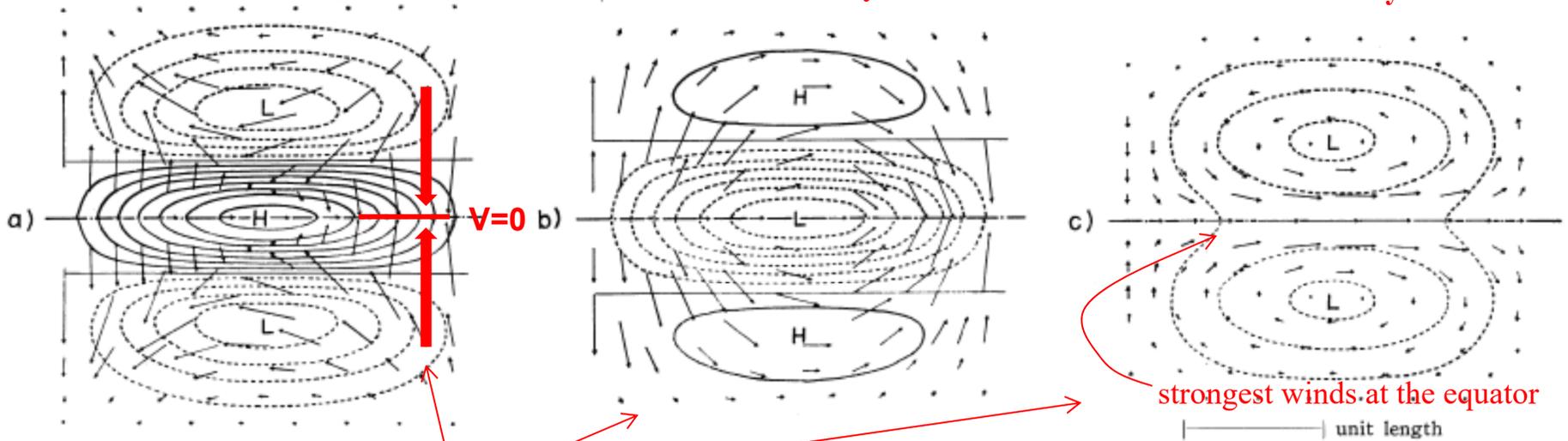


examples

Eastward Gravity Wave

Westward Gravity Wave

Westward Rossby Wave

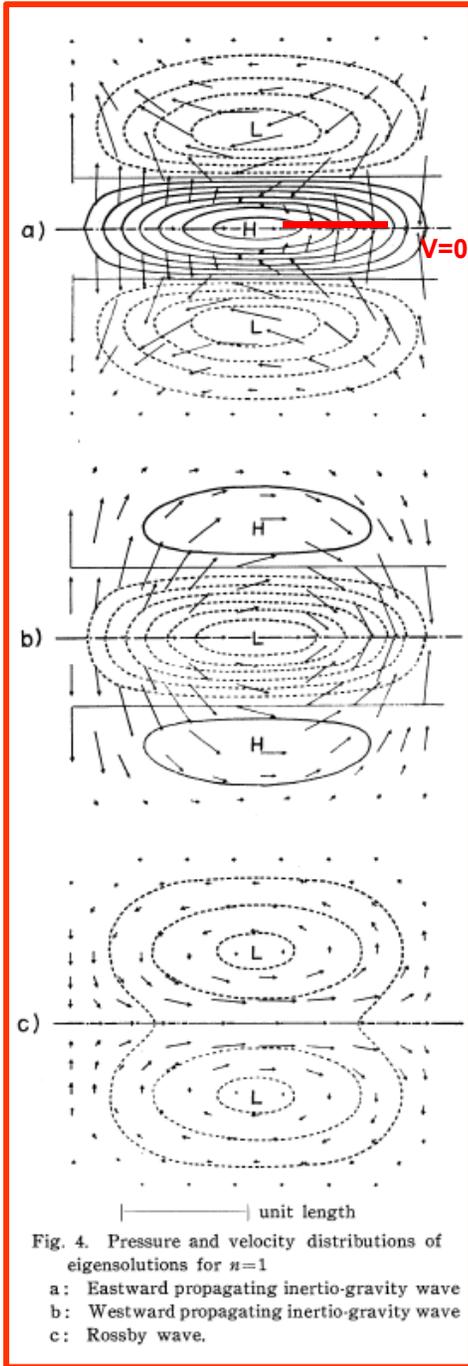


meridional winds (v) change sign once ($n=1$)

(from Matsuno 1966)

Fig. 4. Pressure and velocity distributions of eigensolutions for $n=1$
 a: Eastward propagating inertio-gravity wave
 b: Westward propagating inertio-gravity wave
 c: Rossby wave.

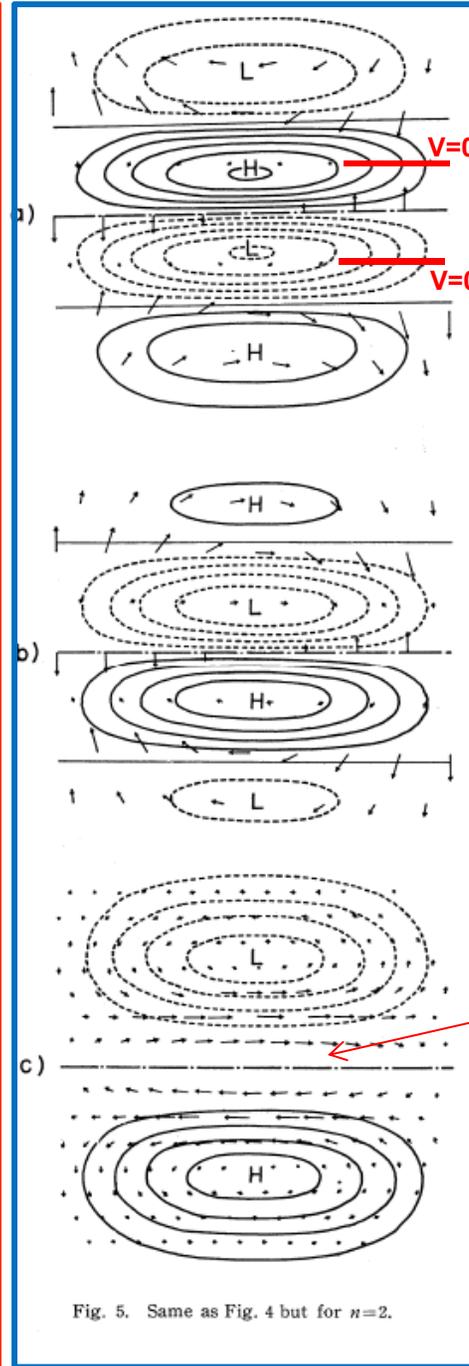
n=1



Symmetric in pressure w.r.t. the equator

(from Matsuno 1966)

n=2



Anti-symmetric in pressure

Counter-clockwise vortex at the equator without a counterpart in the pressure fields
 → due to sign change of Coriolis parameter



Equatorial Waves with $n=0$ (Mixed Rossby-Gravity Waves)

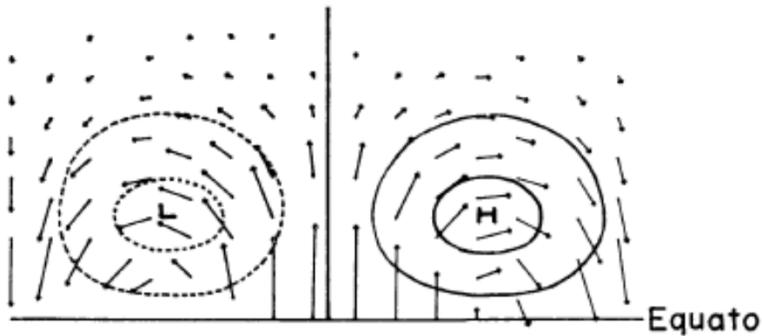
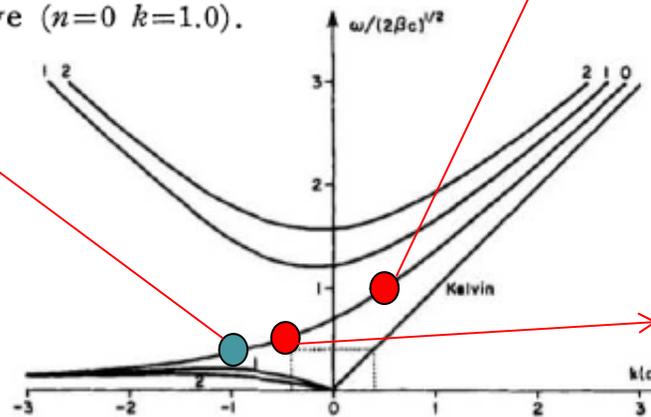


Fig. 7. Rossby type wave ($n=0$ $k=1.0$).



(from Matsuno 1966)

The wave solution with $n=0$ is special

- ➔ Only two solutions exist
- ➔ Which behaves like a Rossby wave for large negative k but behaves like a gravity wave for large positive k .
- ➔ This group of waves is called “mixed Rossby-Gravity Waves”.

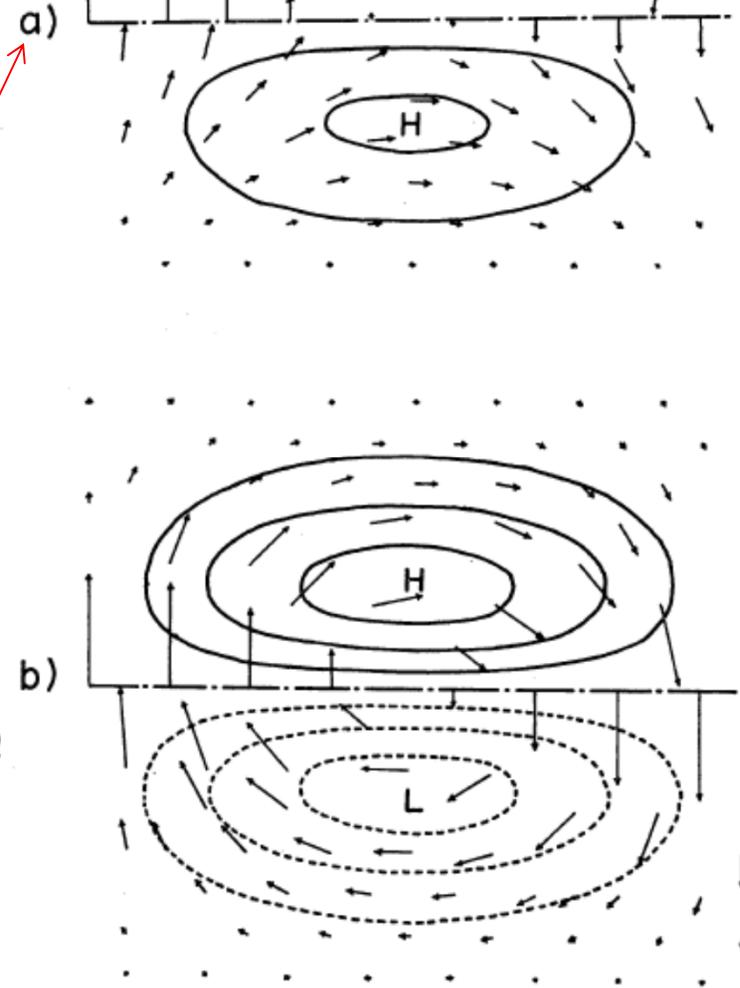


Fig. 6. Pressure and velocity distributions of eigenfunctions for $n=0$ and $k=0.5$
a: Eastward moving inertio-gravity wave
b: Westward moving inertio-gravity wave.

Mixed Rossby-Gravity Waves

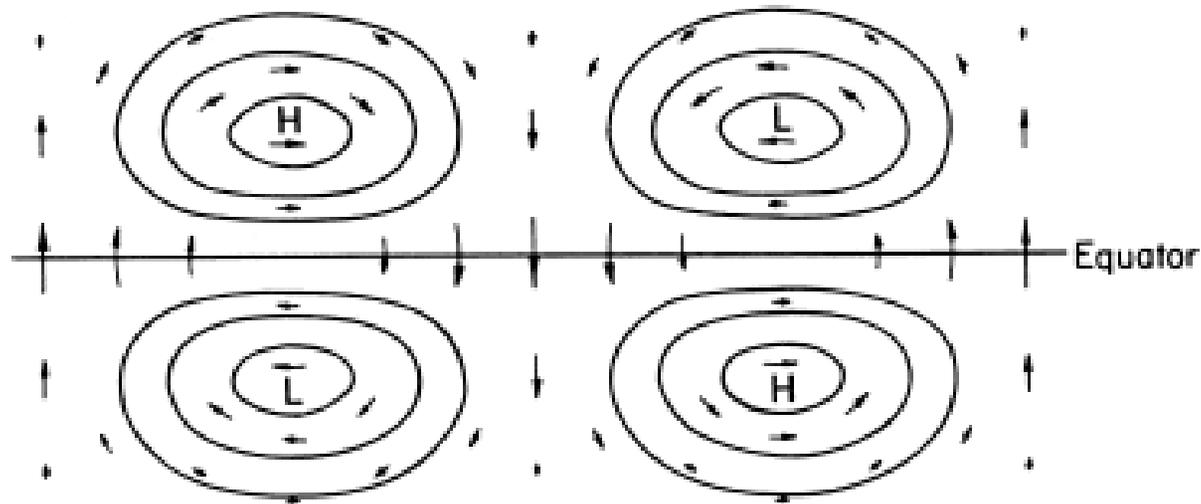
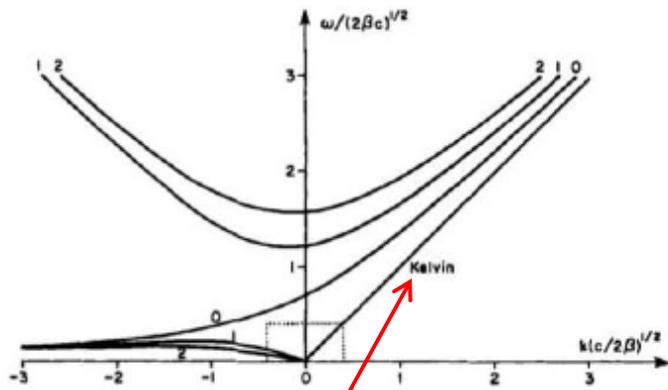


Fig. 11.13 Plan view of horizontal velocity and height perturbations associated with an equatorial Rossby-gravity wave. (Adapted from Matsuno, 1966.)



Equatorial Waves with “ $n = -1$ ”

(Equatorial Kelvin Waves)



(from Matsuno 1966)

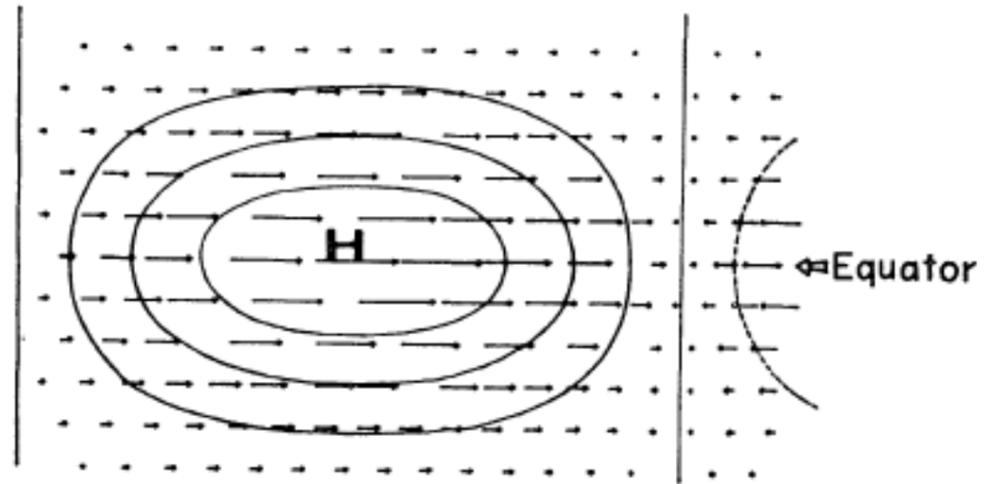


Fig. 8. Pressure and velocity distributions of eigensolution for $n = -1$ and $k = 0.5$. This wave behaves like as the Kelvin wave.

- The equatorially trapped Kelvin wave satisfies the dispersion relationship of the equatorial beta-plane with $n = -1$.
- The solution is obtained by putting $v(x, y) = 0 \rightarrow$ **no meridional wind component**
- The solution of this type behaves like as **a pure gravity wave in the x-direction**, while in the **y-direction the geostrophic relation holds** between zonal velocity and meridional pressure gradient.

Equatorial Kelvin Waves

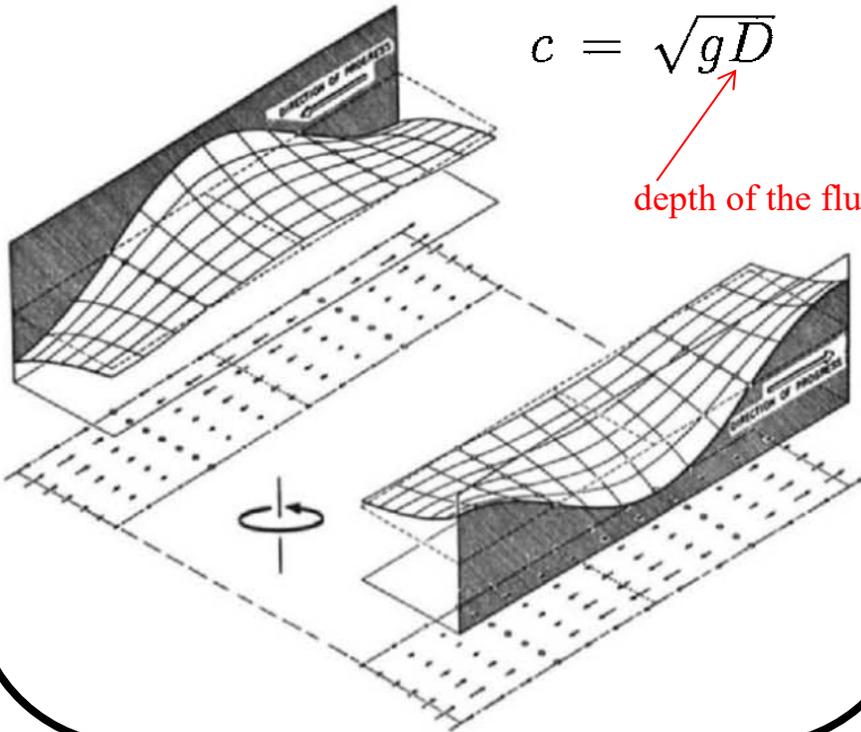
Coastal Kelvin Wave

$$H = \text{const} \times \exp\left(-\frac{f}{c}y\right)$$

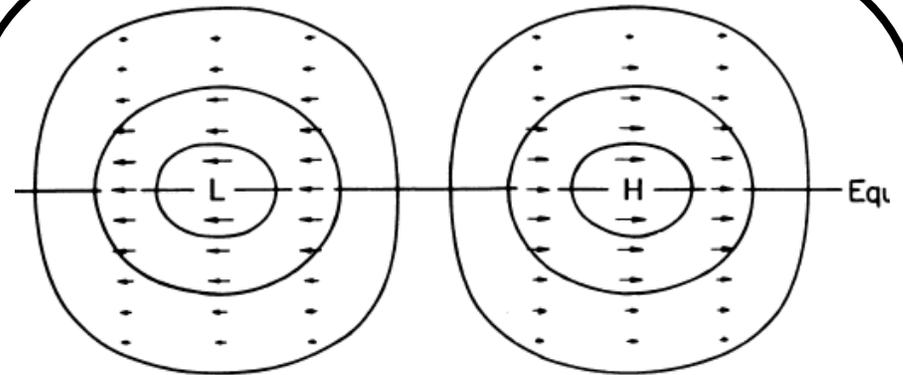
At the coast $y = 0$ is $v = 0$:

$$c = \sqrt{gD}$$

depth of the fluid



Equatorial Kelvin Wave



$$\hat{u} = u_0 \exp\left(-\beta y^2 / 2c\right)$$

$$Y_K = |2c/\beta|^{1/2} \rightarrow \text{e-decaying width}$$

for a phase speed $c = 30\text{ms}^{-1}$
gives $Y_K \approx 1600 \text{ km}$.



Quasi-Biennial Oscillation (QBO) (in Stratosphere)

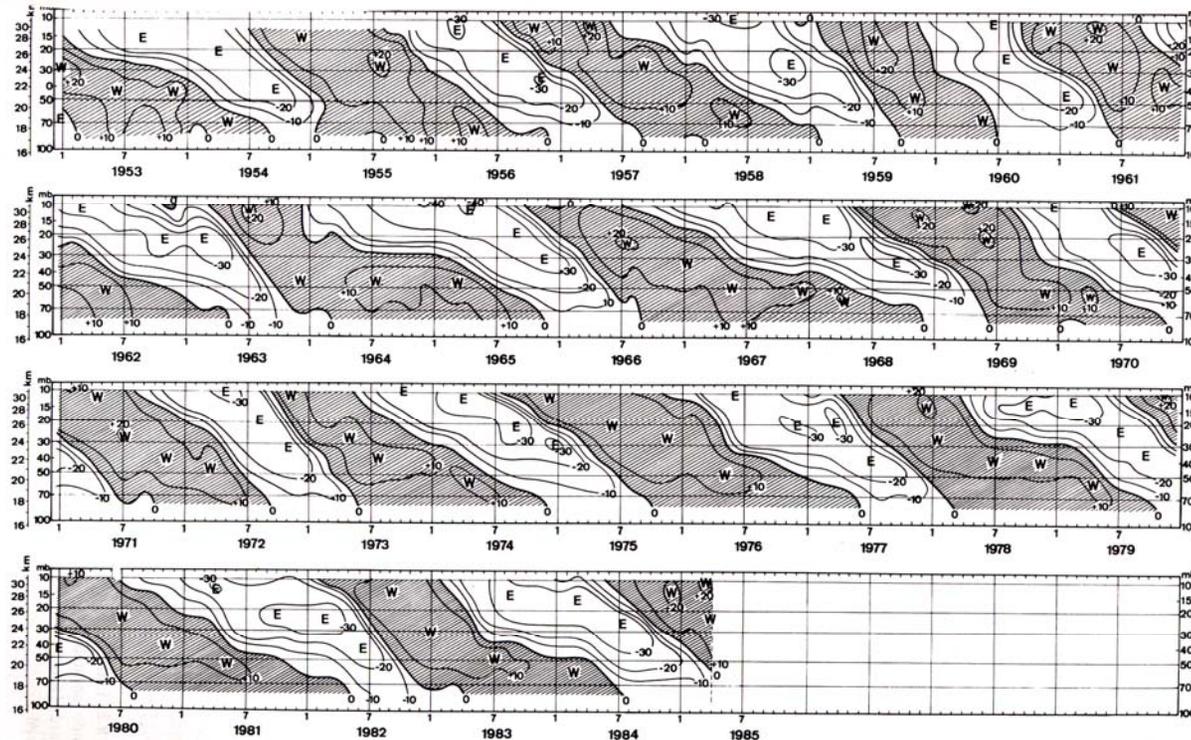


Fig. 8.1. Time-height section of monthly mean zonal winds (m s^{-1}) at equatorial stations (Jan. 1953–Aug. 1967: Canton Island, $3^{\circ}\text{S}/172^{\circ}\text{W}$; Sept. 1967–Dec. 1975: Gan/Maldives Islands, $1^{\circ}\text{S}/73^{\circ}\text{E}$; Jan. 1976–Apr. 1985: Singapore, $1^{\circ}\text{N}/104^{\circ}\text{E}$). Isoleths are at 10-m s^{-1} intervals. Note the alternating downward propagating westerly (W) and easterly (E) regimes. [From Naujokat (1986), with permission.]

- ❑ Quasi-Biennial Oscillation: Easterly and westerly winds alternate every other years (approximately) in the lower to middle parts of the tropical stratosphere.

