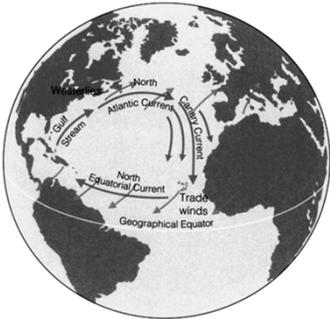


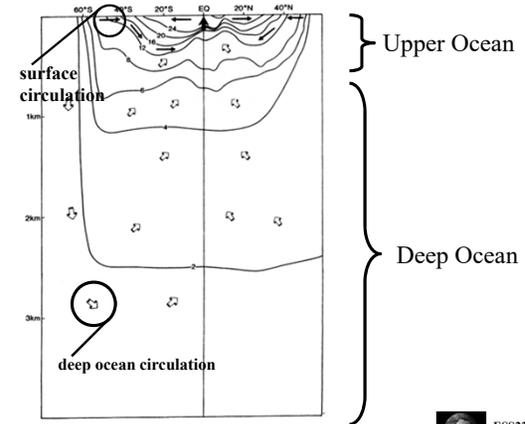
Lecture 10: Ocean Circulation



- ❑ Wind-Driven Circulation
- ❑ Ekman Layer, Transport, Pumping
- ❑ Sverdrup Theory
- ❑ Western Boundary Current

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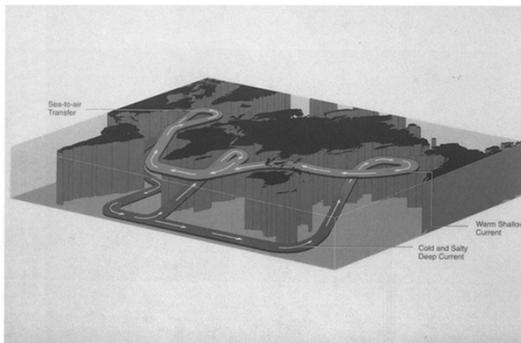
Basic Ocean Current Systems



(from "Is The Temperature Rising?")

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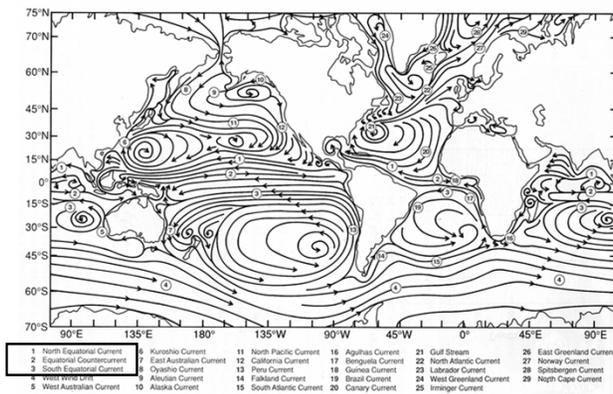
Thermohaline Conveyor Belt



(Figure from Climate System Modeling)

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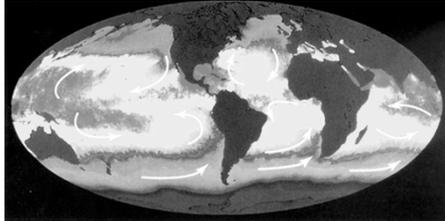
Global Surface Currents



(from Climate System Modeling)

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Six Great Current Circuits in the World Ocean

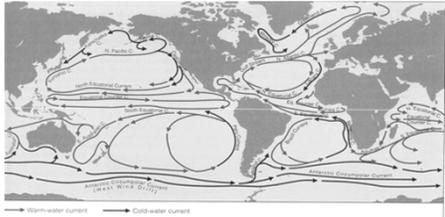


□ 5 of them are geostrophic gyres:

- North Pacific Gyre
- South Pacific Gyre
- North Atlantic Gyre
- South Atlantic Gyre
- Indian Ocean Gyre

□ The 6th and the largest current:

- Antarctic Circumpolar Current
(also called West Wind Drift)

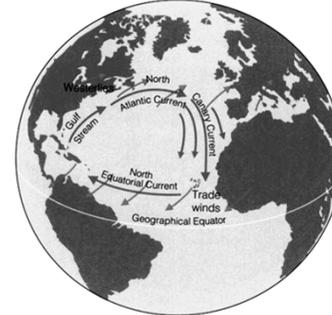


(Figure from *Oceanography* by Tom Garrison)



Characteristics of the Gyres

(Figure from *Oceanography* by Tom Garrison)



□ Currents are in geostrophic balance

□ Each gyre includes 4 current components:
two boundary currents: western and eastern
two transverse currents: eastward and westward

Western boundary current (jet stream of ocean)
the fast, deep, and narrow current moves warm water poleward (transport ~50 Sv or greater)

Eastern boundary current
the slow, shallow, and broad current moves cold water equatorward (transport ~ 10-15 Sv)

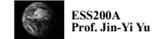
Trade wind-driven current
the moderately shallow and broad westward current (transport ~ 30 Sv)

Westerly-driven current
the wider and slower (than the trade wind-driven current) eastward current

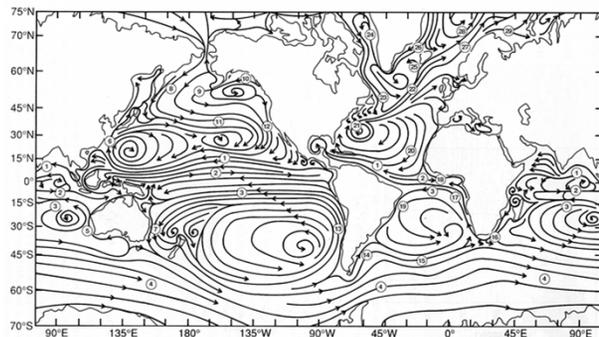
Volume transport unit:

1 sv = 1 Sverdrup = 1 million m³/sec

(the Amazon river has a transport of ~0.17 Sv)

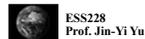


Global Surface Currents

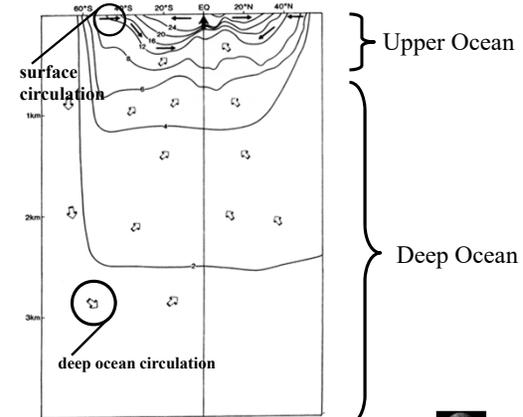


- | | | | | | |
|-----------------------------|---------------------------|---------------------------|---------------------|---------------------------|---------------------------|
| 1 North Equatorial Current | 6 Kuroshio Current | 11 North Pacific Current | 16 Agulhas Current | 21 Gulf Stream | 26 East Greenland Current |
| 2 Equatorial Countercurrent | 7 East Australian Current | 12 California Current | 17 Benguela Current | 22 North Atlantic Current | 27 Norway Current |
| 3 South Equatorial Current | 8 Oyashio Current | 13 Peru Current | 18 Guinea Current | 23 Labrador Current | 28 Spitzbergen Current |
| 4 West Wind Drift | 9 Alaskan Current | 14 Falkland Current | 19 Brazil Current | 24 West Greenland Current | 29 North Cape Current |
| 5 West Australian Current | 10 Alaska Current | 15 South Atlantic Current | 20 Canary Current | 25 Inringer Current | |

(from *Climate System Modeling*)



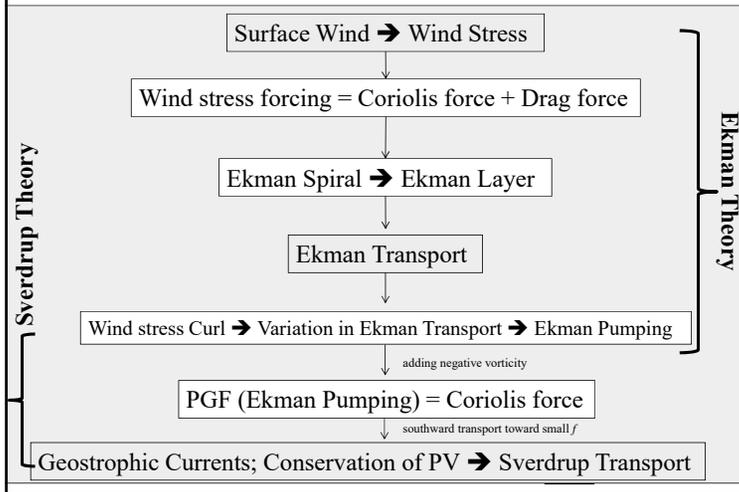
Basic Ocean Current Systems



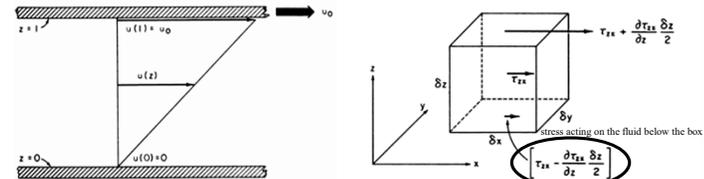
(from "*Is The Temperature Rising?*")



Wind-Driven (Upper Ocean) Circulation

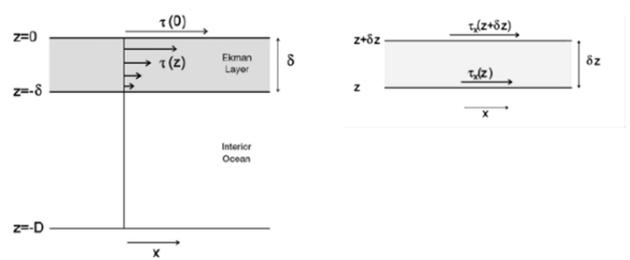


Frictional (Viscous) Force



- Any real fluid is subject to internal friction (viscosity)
- Force required to maintain this flow
 $\rightarrow F = \mu A u_0 / l$
- For a layer of fluid at depth δz , the force is
 $\rightarrow F = \mu A \delta u / \delta z$
- Viscous force per unit area (shearing stress):
 $\rightarrow \tau_{zx} = \lim_{\delta z \rightarrow 0} \mu \frac{\delta u}{\delta z} = \mu \frac{\partial u}{\partial z}$
- Stresses applied on a fluid element
 $\rightarrow \left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} \delta z \right) \delta x \delta y - \left(\tau_{zx} - \frac{\partial \tau_{zx}}{\partial z} \delta z \right) \delta x \delta y$
- Viscous force per unit mass due to stress
 $\rightarrow \frac{1}{\rho} \frac{\partial \tau_{zx}}{\partial z} = \frac{1}{\rho} \frac{\partial}{\partial z} \left(\mu \frac{\partial u}{\partial z} \right)$
- Frictional force per unit mass in x-direction
 $\rightarrow F_{rx} = \nu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right]$
 where $\nu = \mu / \rho$

Surface Wind Stress



Surface wind stress: $(\tau_{wind_x}, \tau_{wind_y}) = \rho_{air} C_D U_{10} (u_a, v_a)$

$$F_x = \frac{\text{force per unit area}}{\text{mass per unit area}} = \frac{\tau_x(z + \delta z) - \tau_x(z)}{\rho_{ref} \delta z} = \frac{1}{\rho_{ref}} \frac{\partial \tau_x}{\partial z}$$

$$F = \frac{1}{\rho_{ref}} \frac{\partial \tau}{\partial z}$$

(from John Marshall and R. Alan Plumb's Atmosphere, Ocean and Climate Dynamics: An Introductory Text)

Step 1: Surface Winds

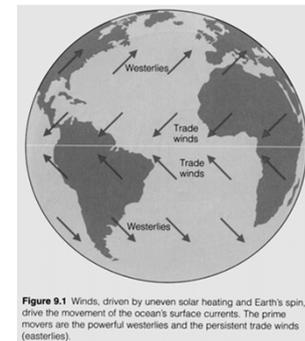


Figure 9.1 Winds, driven by uneven solar heating and Earth's spin, drive the movement of the ocean's surface currents. The prime movers are the powerful westerlies and the persistent trade winds (easterlies).

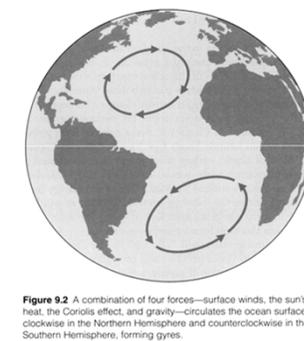
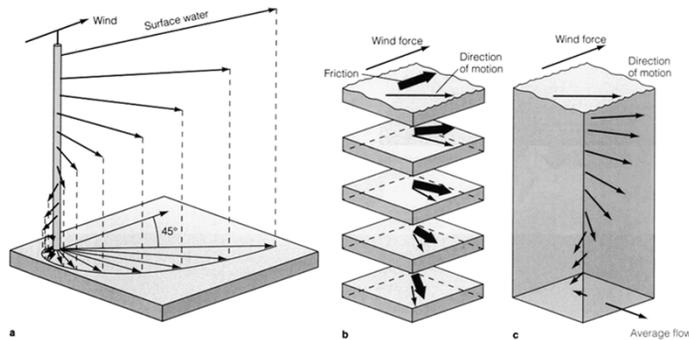


Figure 9.2 A combination of four forces—surface winds, the sun's heat, the Coriolis effect, and gravity—circulates the ocean surface clockwise in the Northern Hemisphere and counterclockwise in the Southern Hemisphere, forming gyres.

(Figure from Oceanography by Tom Garrison)

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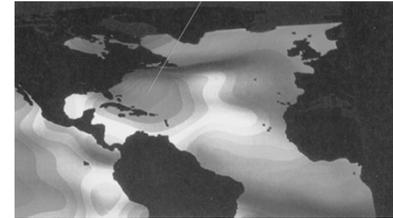
Step 2: Ekman Layer (frictional force + Coriolis Force)



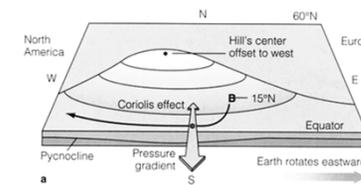
(Figure from *Oceanography* by Tom Garrison)

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Step 3: Geostrophic Current (Pressure Gradient Force + Coriolis Force)



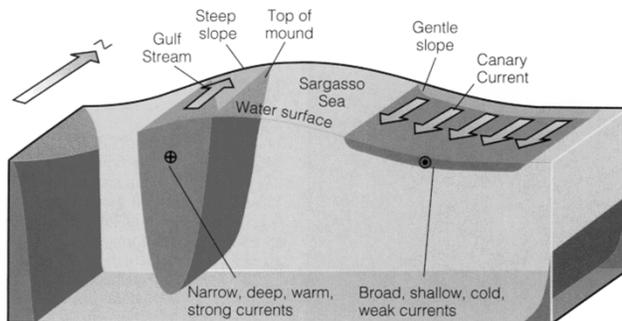
NASA-TOPEX
Observations of
Sea-Level Height



(from *Oceanography* by Tom Garrison)

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Step 4: Boundary Currents



(Figure from *Oceanography* by Tom Garrison)

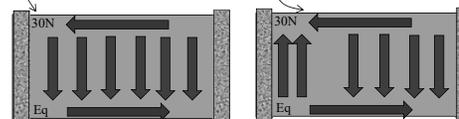
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History / Wind-Driven Circulation

(from Robert H. Stewart's book on "Introduction to Physical Oceanography")

Table 9.2 Contributions to the Theory of the Wind-Driven Circulation

Contributor	Year	Contribution
Fridtjof Nansen	(1898)	Qualitative theory, currents transport water at an angle to the wind.
Vagn Walfrid Ekman	(1902)	Quantitative theory for wind-driven transport at the sea surface.
Harald Sverdrup	(1947)	Theory for wind-driven circulation in the eastern Pacific.
Henry Stommel	(1948)	Theory for westward intensification of wind-driven circulation (western boundary currents).
Walter Munk	(1950)	Quantitative theory for main features of the wind-driven circulation.
Kirk Bryan	(1963)	Numerical models of the oceanic circulation.
Bert Semtner and Robert Chervin	(1988)	Global, eddy-resolving, realistic model of the ocean's circulation.



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Step 1: Surface Winds

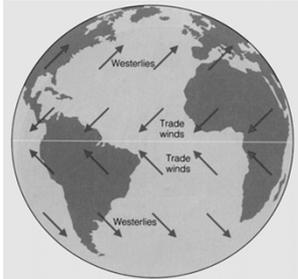


Figure 9.1 Winds, driven by uneven solar heating and Earth's spin, drive the movement of the ocean's surface currents. The prime movers are the powerful westerlies and the persistent trade winds (easterlies).

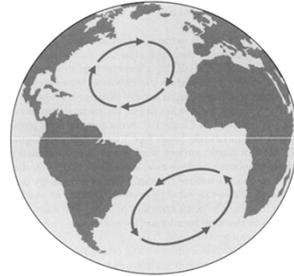


Figure 9.2 A combination of four forces—surface winds, the sun's heat, the Coriolis effect, and gravity—circulates the ocean surface clockwise in the Northern Hemisphere and counterclockwise in the Southern Hemisphere, forming gyres.

(Figure from *Oceanography* by Tom Garrison)



Why an Angle btw Wind and Iceberg Directions?

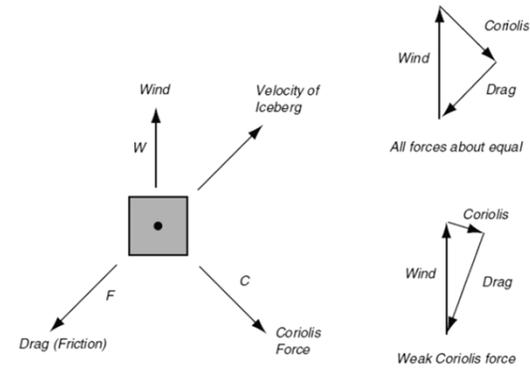
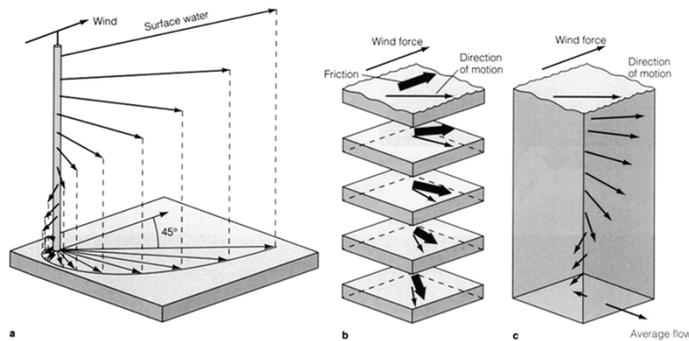


Figure 9.2 The balance of forces acting on an iceberg in a wind on a rotating Earth.

(from Robert H. Stewart's book on "Introduction to Physical Oceanography")

Step 2: Ekman Layer (frictional force + Coriolis Force)



(Figure from *Oceanography* by Tom Garrison)



In the Boundary Layer

For a steady state, homogeneous boundary layer

$$fv + A_z \frac{\partial^2 u}{\partial z^2} = 0$$

$$-fu + A_z \frac{\partial^2 v}{\partial z^2} = 0$$

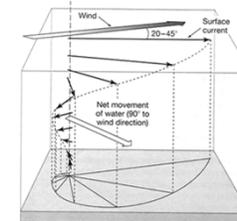
viscosity

Coriolis force balances frictional force

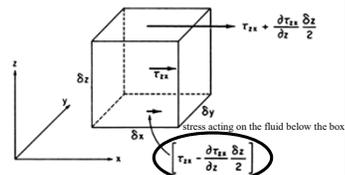
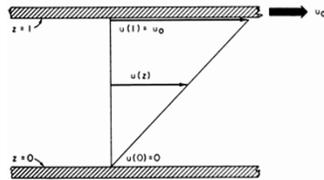
$$u = V_0 \exp(az) \cos(\pi/4 + az)$$

$$v = V_0 \exp(az) \sin(\pi/4 + az)$$

$$a = \sqrt{\frac{f}{2A_z}}$$



Frictional (Viscous) Force

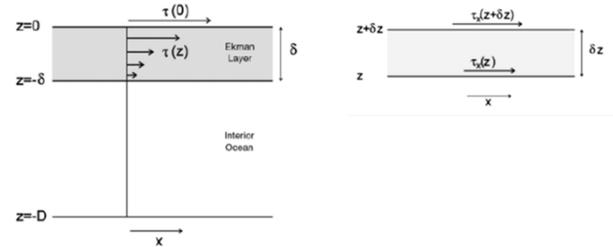


- Any real fluid is subject to internal friction (viscosity)
- Force required to maintain this flow
 $\rightarrow F = \mu A u_0 / l$
- For a layer of fluid at depth δz , the force is
 $\rightarrow F = \mu A \delta u / \delta z$
- Viscous force per unit area (shearing stress):
 $\rightarrow \tau_{zx} = \lim_{\delta z \rightarrow 0} \mu \frac{\delta u}{\delta z} = \mu \frac{\partial u}{\partial z}$

- Stresses applied on a fluid element
 $\rightarrow \left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} \frac{\delta z}{2} \right) \delta x \delta y - \left(\tau_{zx} - \frac{\partial \tau_{zx}}{\partial z} \frac{\delta z}{2} \right) \delta x \delta y$
- Viscous force per unit mass due to stress
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- Frictional force per unit mass in x-direction
 $\rightarrow F_{rx} = v \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right]$
 where $v = \mu / \rho$

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Surface Wind Stress



Surface wind stress: $(\tau_{wind_x}, \tau_{wind_y}) = \rho_{air} C_D U_{10} (u_a, v_a)$

$$F_x = \frac{\text{force per unit area}}{\text{mass per unit area}} = \frac{\tau_x(z + \delta z) - \tau_x(z)}{\rho_{ref} \delta z} = \frac{1}{\rho_{ref}} \frac{\partial \tau_x}{\partial z}$$

$$F = \frac{1}{\rho_{ref}} \frac{\partial \tau}{\partial z}$$

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(from John Marshall and R. Alan Plumb's Atmosphere, Ocean and Climate Dynamics: An Introductory Text)

How Deep is the Ekman Layer?

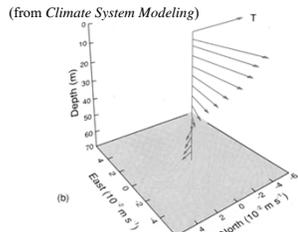


Fig. 4.4 (a) Vertical distribution of temperature and salinity at 50°N, 145°W, in early September, 1977. The solid lines are before a storm and the dotted lines are after a storm, which depict the vertical mixing above the seasonal thermocline. The main thermocline, or pycnocline in this area is between 110 m and 160 m depth. (b) Time-averaged velocity for a 25 day summer period at an open ocean site southwest of Bermuda. Current meter measured velocity is referenced to 70 m. The topmost dashed vector is the time-averaged wind stress (Price et al., 1986).

$$D \propto (v/f)^{1/2}$$

v = vertical diffusivity of momentum

f = Coriolis parameter = $2\Omega \sin\phi$

The thickness of the Ekman layer is arbitrary because the Ekman currents decrease exponentially with depth. Ekman proposed that the thickness be the depth D_E at which the current velocity is opposite the velocity at the surface, which occurs at a depth $D_E = \pi/a$

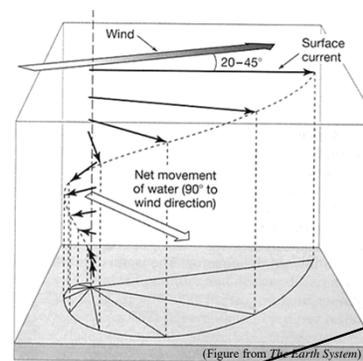
$$D_E = \sqrt{\frac{2\pi^2 A_z}{f}}$$

(from Robert H. Steward)



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Ekman Transport



$$fv + A_z \frac{\partial^2 u}{\partial z^2} = 0$$

$$-fu + A_z \frac{\partial^2 v}{\partial z^2} = 0$$

or

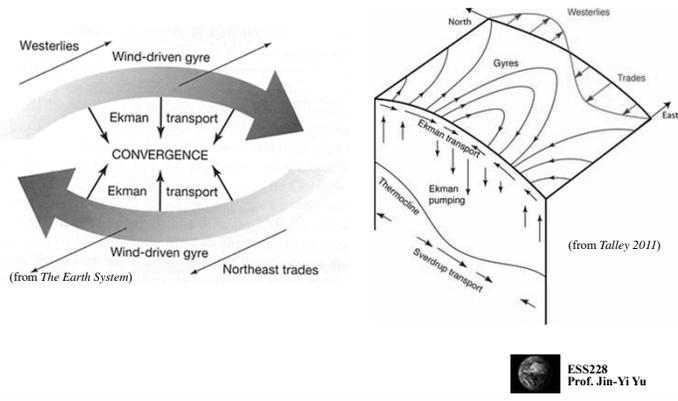
$$\rho f V + \frac{\partial T_{xz}}{\partial z} = 0$$

$$\rho f U - \frac{\partial T_{yz}}{\partial z} = 0$$

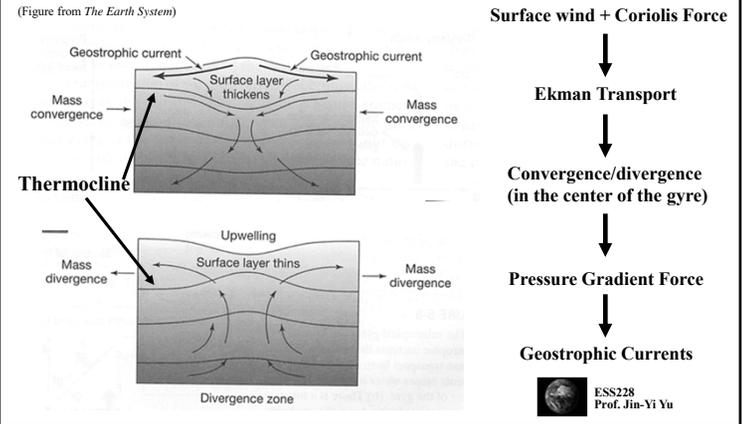
$$U_E = \int_{-\infty}^0 u_E dz = \frac{\tau_y}{\rho_o f};$$

$$V_E = \int_{-\infty}^0 v_E dz = -\frac{\tau_x}{\rho_o f}$$

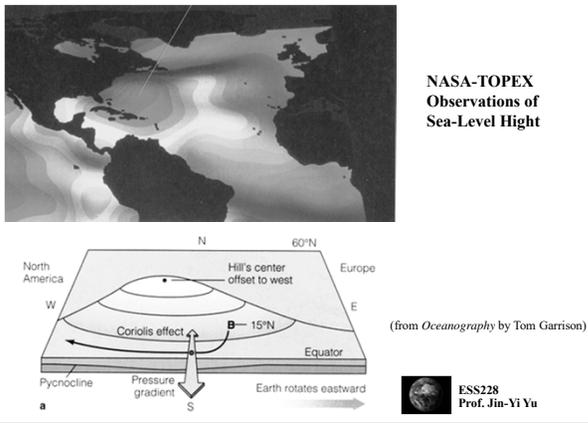
Ekman Transport and Ekman Pumping



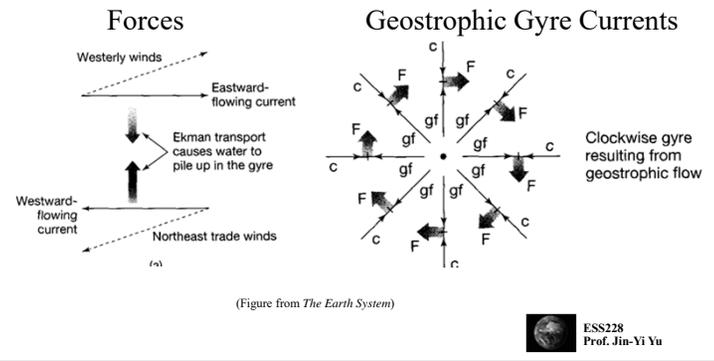
Ekman Transport → Convergence/Divergence



Step 3: Geostrophic Current (Pressure Gradient Force + Coriolis Force)

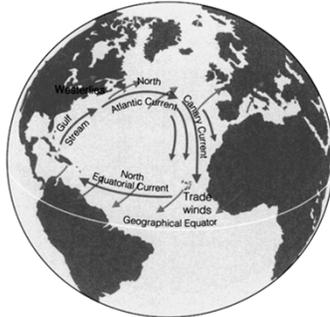


Geostrophic Current



Characteristics of the Gyres

(Figure from *Oceanography* by Tom Garrison)



Volume transport unit:

1 sv = 1 Sverdrup = 1 million m³/sec
(the Amazon river has a transport of ~0.17 Sv)

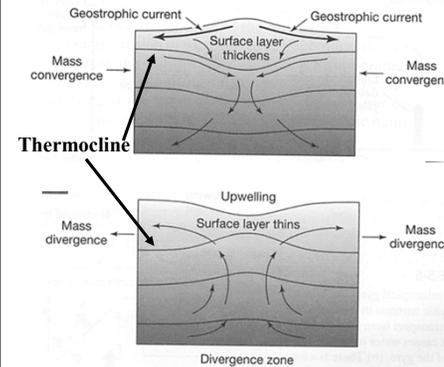
- **Currents are in geostrophic balance**
- **Each gyre includes 4 current components:**
 - two boundary currents: western and eastern
 - two transverse currents: eastward and westward
- Western boundary current (jet stream of ocean)**
the fast, deep, and narrow current moves **warm** water poleward (transport ~50 Sv or greater)
- Eastern boundary current**
the slow, shallow, and broad current moves **cold** water equatorward (transport ~ 10-15 Sv)
- Trade wind-driven current**
the moderately shallow and broad westward current (transport ~ 30 Sv)
- Westerly-driven current**
the wider and slower (than the trade wind-driven current) eastward current



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Ekman Transport → Convergence/Divergence

(Figure from *The Earth System*)



Surface wind + Coriolis Force

Ekman Transport

Convergence/divergence
(in the center of the gyre)

Pressure Gradient Force

Geostrophic Currents



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Theories that Explain the Wind-Driven Ocean Circulation

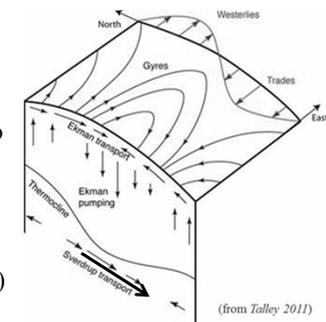
- Harald Sverdrup (1947) showed that the circulation in the upper kilometer or so of the ocean is directly related to the curl of the wind stress if the Coriolis force varies with latitude.
- Henry Stommel (1948) showed that the circulation in oceanic gyres is asymmetric also because the Coriolis force varies with latitude.
- Walter Munk (1950) added eddy viscosity and calculated the circulation of the upper layers of the Pacific.
- Together the three oceanographers laid the foundations for a modern theory of ocean circulation.

(from Robert H. Stewart's book on "*Introduction to Physical Oceanography*")

Sverdrup's Theory of the Oceanic Circulation

$$V = \hat{k} \cdot \frac{\nabla \times \tau}{\beta}$$

- The Sverdrup balance, or Sverdrup relation, is a theoretical relationship between the wind stress exerted on the surface of the open ocean and the vertically integrated meridional (north-south) transport of ocean water.

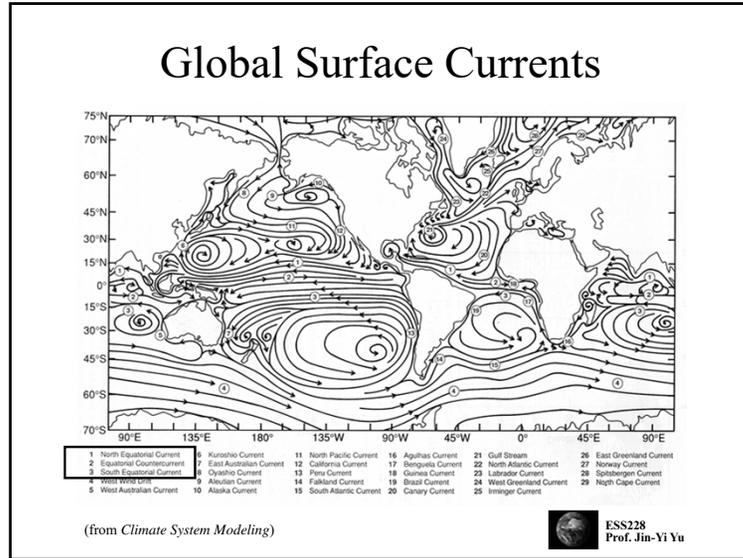


(from Talley 2011)

- Positive wind stress curl → Northward mass transport
- Negative wind stress curl → Southward mass transport



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Sverdrup Transport

$$\frac{\partial p}{\partial x} = f \rho v + \frac{\partial T_{xz}}{\partial z} \quad \frac{\partial p}{\partial y} = -f \rho u + \frac{\partial T_{yz}}{\partial z}$$

$$\frac{\partial P}{\partial x} = \int_{-D}^0 \frac{\partial p}{\partial x} dz, \quad \frac{\partial P}{\partial y} = \int_{-D}^0 \frac{\partial p}{\partial y} dz,$$

$$M_x = \int_{-D}^0 \rho u(z) dz, \quad M_y = \int_{-D}^0 \rho v(z) dz,$$

$$\frac{\partial P}{\partial x} = f M_y + T_x$$

$$\frac{\partial P}{\partial y} = -f M_x + T_y$$

vertical integration from surface (z=0) to a depth of no motion (z=-D).

$$d/dy \left(\frac{\partial P}{\partial x} = f M_y + T_x \right) - d/dx \left(\frac{\partial P}{\partial y} = -f M_x + T_y \right) \text{ and use } \frac{\partial M_x}{\partial x} + \frac{\partial M_y}{\partial y} = 0$$

$$\beta M_y = \frac{\partial T_y}{\partial x} - \frac{\partial T_x}{\partial y}$$

$$\beta M_y = \text{curl}_z(T)$$

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Sverdrup, Geostrophic, and Ekman Transports

$$V = \hat{k} \cdot \frac{\nabla \times \tau}{\beta}$$

- Continuity equation for an incompressible flow:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$
- Assume the horizontal flows are geostrophic:

$$\frac{\partial u_z}{\partial x} + \frac{\partial v_z}{\partial x} + \frac{\partial w}{\partial z} = 0$$
- Replace the geostrophic flow pressure gradients:

$$f u_z = -\frac{1}{\rho} \frac{\partial P}{\partial y}$$

$$f v_z = \frac{1}{\rho} \frac{\partial P}{\partial x}$$
- The continuity equation becomes:

$$-\frac{\beta}{f} v_z + \frac{\partial w}{\partial z} = 0 \Rightarrow \beta v_z = f \frac{\partial w}{\partial z}$$

Ekman layer pumping
→ vertical depth decreases
→ move equatorward to conserve absolute vorticity.

Ekman layer suction
→ vertical depth increases
→ move poleward to conserve absolute vorticity.

$$(\zeta + f)/h = \eta/h = \text{Const}$$

Sverdrup, Geostrophic, and Ekman Transports

$$V = \hat{k} \cdot \frac{\nabla \times \tau}{\beta}$$

$$V_E = \int_{-\infty}^0 v_E dz = -\frac{\tau_x}{\rho_o f}$$

- Continuity equation for an incompressible flow:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$
- Assume the horizontal flows are geostrophic:

$$\frac{\partial u_z}{\partial x} + \frac{\partial v_z}{\partial x} + \frac{\partial w}{\partial z} = 0$$
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$$f u_z = -\frac{1}{\rho} \frac{\partial P}{\partial y}$$

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Ekman pumping
→ vertical depth decreases
→ move equatorward to conserve absolute vorticity.

Integrate the equation from the bottom of the upper ocean (D_w) to the bottom of the Ekman layer (D_E):

$$\beta \int_{z=D_w}^{z=D_E} v dz = f [w_E - w(-D_w)] \text{ assume zero}$$

Ekman pumping (W_E) is related to the convergence of the Ekman transport:

$$w(-D_E) = \frac{\partial}{\partial x} \left(\frac{\tau^y}{\rho f} \right) - \frac{\partial}{\partial y} \left(\frac{\tau^x}{\rho f} \right)$$

Therefore, we obtain:

$$\int_{z=D_w}^{z=D_E} v dz = \frac{1}{\rho \beta} \left(\frac{\partial \tau^y}{\partial x} - \frac{\partial \tau^x}{\partial y} \right) + \frac{1}{\rho f} \tau_w^x$$

geostrophic transport Sverdrup transport - (Ekman Transport)

Therefore,
Sverdrup transport = Geostrophic transport + Ekman transport

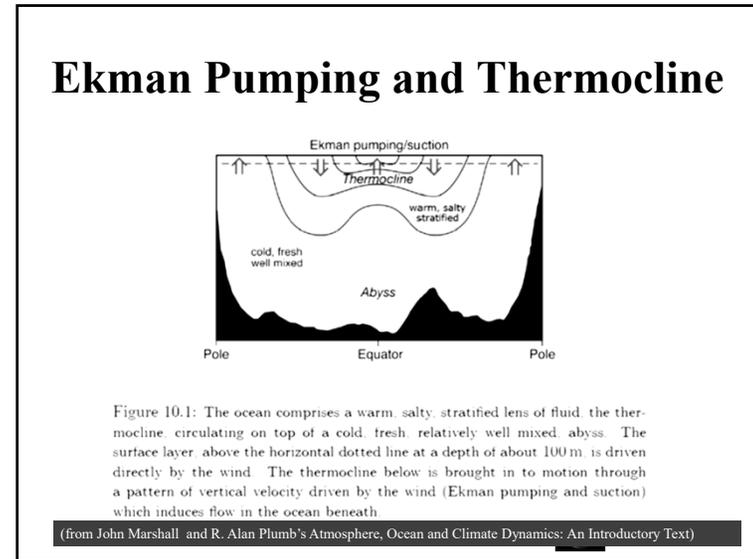
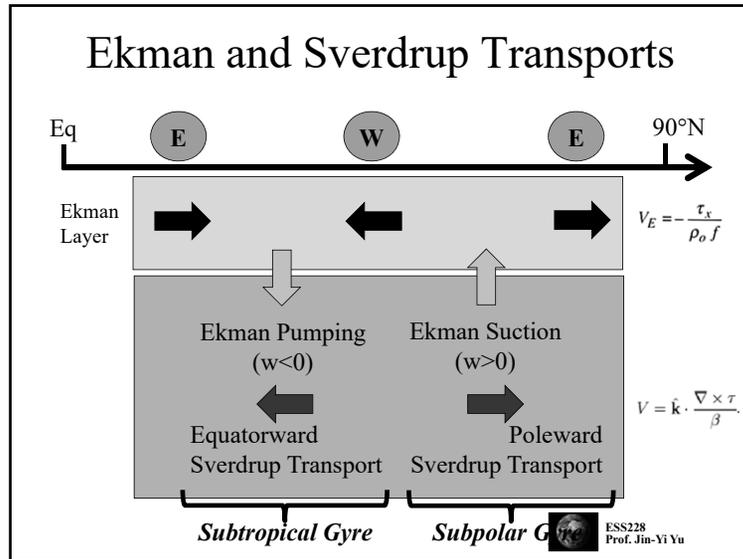
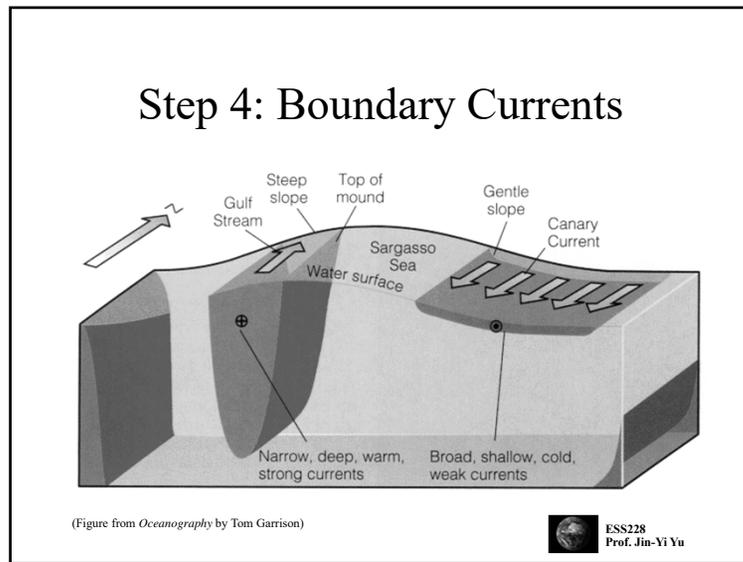


Figure 10.1: The ocean comprises a warm, salty, stratified lens of fluid, the thermocline, circulating on top of a cold, fresh, relatively well mixed, abyss. The surface layer, above the horizontal dotted line at a depth of about 100 m, is driven directly by the wind. The thermocline below is brought in to motion through a pattern of vertical velocity driven by the wind (Ekman pumping and suction) which induces flow in the ocean beneath.

(from John Marshall and R. Alan Plumb's *Atmosphere, Ocean and Climate Dynamics: An Introductory Text*)



(Figure from *Oceanography* by Tom Garrison)

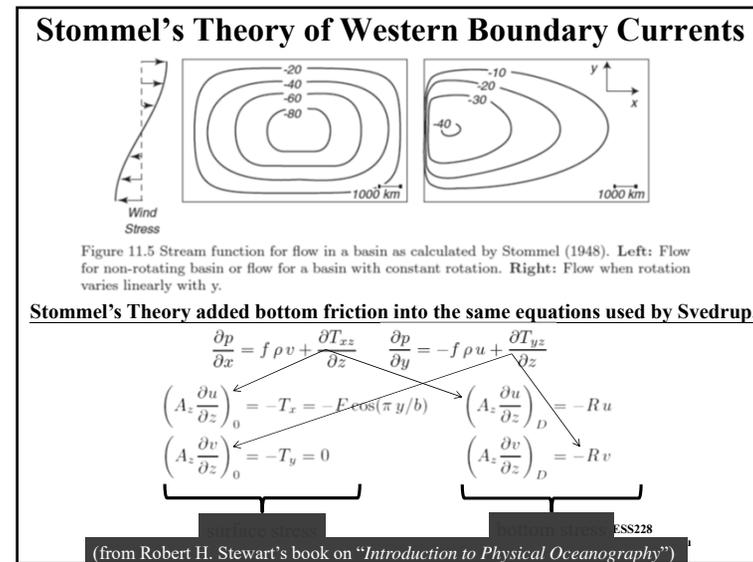


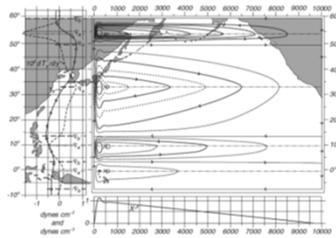
Figure 11.5 Stream function for flow in a basin as calculated by Stommel (1948). Left: Flow for non-rotating basin or flow for a basin with constant rotation. Right: Flow when rotation varies linearly with y .

Stommel's Theory added bottom friction into the same equations used by Sverdrup.

$$\begin{aligned} \frac{\partial p}{\partial x} &= f \rho v + \frac{\partial T_{xz}}{\partial z} & \frac{\partial p}{\partial y} &= -f \rho u + \frac{\partial T_{yz}}{\partial z} \\ \left(A_z \frac{\partial u}{\partial z} \right)_0 &= -T_x = -E \cos(\pi y/b) & \left(A_z \frac{\partial u}{\partial z} \right)_D &= -R u \\ \left(A_z \frac{\partial v}{\partial z} \right)_0 &= -T_y = 0 & \left(A_z \frac{\partial v}{\partial z} \right)_D &= -R v \end{aligned}$$

(from Robert H. Stewart's book on "Introduction to Physical Oceanography")

Munk's Theory of Western Boundary Currents



$$\frac{1}{\rho} \frac{\partial p}{\partial x} = f v + \frac{\partial}{\partial z} \left(A_z \frac{\partial u}{\partial z} \right) + A_H \frac{\partial^2 u}{\partial x^2} + A_H \frac{\partial^2 u}{\partial y^2}$$

$$\frac{1}{\rho} \frac{\partial p}{\partial y} = -f u + \frac{\partial}{\partial z} \left(A_z \frac{\partial v}{\partial z} \right) + A_H \frac{\partial^2 v}{\partial x^2} + A_H \frac{\partial^2 v}{\partial y^2}$$

surface stress
lateral friction

$$A_H \nabla^4 \Psi - \beta \frac{\partial \Psi}{\partial x} = -\text{curl}_z T$$

Friction
Sverdrup Balance

mass-transport stream function Ψ

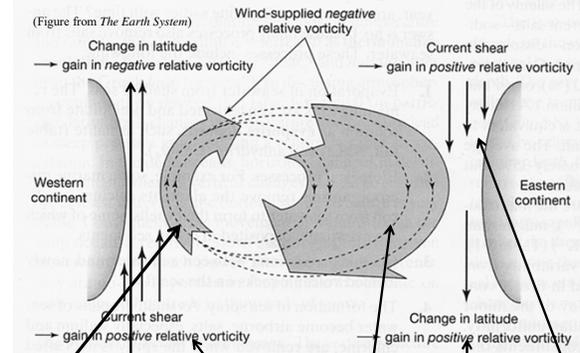
$$M_x \equiv \frac{\partial \Psi}{\partial y}, \quad M_y \equiv -\frac{\partial \Psi}{\partial x}$$

- Munk (1950) built upon Sverdrup's theory, adding lateral eddy viscosity, to obtain a solution for the circulation within an ocean basin.
- To simplify the equations, Munk used the mass-transport stream function.

(from Robert H. Stewart's book on "Introduction to Physical Oceanography")

Why Strong Boundary Currents?

A Potential Vorticity View



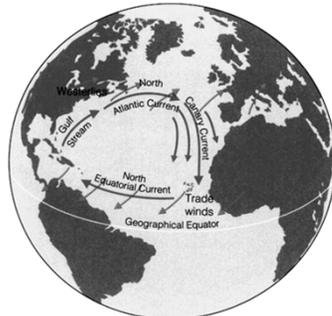
Goal: Maintain the "steady state" of the negative vorticity induced by wind stress curve

$$\xi^- = \xi^- \text{ plus } \xi^+ \quad \xi^- = \xi^+ \text{ plus } \xi^+$$

friction has to be big → strong boundary current

Characteristics of the Gyres

(Figure from *Oceanography* by Tom Garrison)



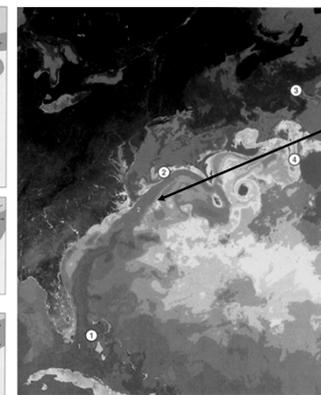
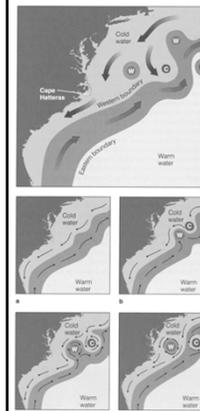
Volume transport unit:

1 sv = 1 Sverdrup = 1 million m³/sec
(the Amazon river has a transport of ~0.17 Sv)

- Currents are in geostrophic balance
- Each gyre includes 4 current components:
 - two boundary currents: western and eastern
 - two transverse currents: eastward and westward
- Western boundary current (jet stream of ocean)**
the fast, deep, and narrow current moves warm water poleward (transport ~50 Sv or greater)
- Eastern boundary current**
the slow, shallow, and broad current moves cold water equatorward (transport ~10-15 Sv)
- Trade wind-driven current**
the moderately shallow and broad westward current (transport ~30 Sv)
- Westerly-driven current**
the wider and slower (than the trade wind-driven current) eastward current

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Gulf Stream



A river of current
Jet stream in the ocean

- Speed = 2 m/sec
- Depth = 450 m
- Width = 70 Km
- Color: clear and blue

(Figure from *Oceanography* by Tom Garrison)

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