Lecture 10: Ocean Circulation

- Wind-Driven Circulation
- Ekman Layer, Transport, Pumping
- Sverdrup Theory
- Western Boundary Current
Basic Ocean Current Systems

(from “Is The Temperature Rising?”)
Thermohaline Conveyor Belt

(Figure from Climate System Modeling)
Global Surface Currents

(from Climate System Modeling)
Six Great Current Circuits in the World Ocean

- **5 of them are geostrophic gyres:**
  - North Pacific Gyre
  - South Pacific Gyre
  - North Atlantic Gyre
  - South Atlantic Gyre
  - Indian Ocean Gyre

- **The 6th and the largest current:**
  - Antarctic Circumpolar Current
    (also called West Wind Drift)

(Figure from *Oceanography* by Tom Garrison)
Characteristics of the Gyres

- **Currents are in geostrophic balance**
- Each gyre includes 4 current components:
  - two boundary currents: western and eastern
  - two transverse currents: eastward and westward

**Western boundary current (jet stream of ocean)**
the fast, deep, and narrow current moves **warm** water polarward (transport ~50 Sv or greater)

**Eastern boundary current**
the slow, shallow, and broad current moves cold water equatorward (transport ~ 10-15 Sv)

**Trade wind-driven current**
the moderately shallow and broad westward current (transport ~ 30 Sv)

**Westerly-driven current**
the wider and slower (than the trade wind-driven current) eastward current

Volume transport unit:
1 sv = 1 Sverdrup = 1 million m³/sec
(the Amazon river has a transport of ~0.17 Sv)
Global Surface Currents

(from Climate System Modeling)
Basic Ocean Current Systems

(from “Is The Temperature Rising?”)
Wind-Driven (Upper Ocean) Circulation

- Surface Wind $\rightarrow$ Wind Stress
- Wind stress forcing = Coriolis force + Drag force
- Ekman Spiral $\rightarrow$ Ekman Layer
- Ekman Transport
- Wind stress Curl $\rightarrow$ Variation in Ekman Transport $\rightarrow$ Ekman Pumping
  - Adding negative vorticity
  - PGF (Ekman Pumping) = Coriolis force
  - Southward transport toward small $f$
- Geostrophic Currents; Conservation of PV $\rightarrow$ Sverdrup Transport
Frictional (Viscous) Force

• Any real fluid is subject to internal friction (viscosity)

• Force required to maintain this flow
  \[ F = \mu Au_0 / l \]

• For a layer of fluid at depth \( \delta Z \), the force is
  \[ F = \mu A \delta u / \delta z \]

• Viscous force per unit area (shearing stress):
  \[ \tau_{zx} = \lim_{\delta z \to 0} \mu \frac{\delta u}{\delta z} = \mu \frac{\partial u}{\partial z} \]

• Stresses applied on a fluid element
  \[ \left( \tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} \frac{\delta z}{2} \right) \delta x \delta y - \left( \tau_{zx} - \frac{\partial \tau_{zx}}{\partial z} \frac{\delta z}{2} \right) \delta x \delta y \]

• Viscous force per unit mass due to stress
  \[ \frac{1}{\rho} \frac{\partial \tau_{zx}}{\partial z} = \frac{1}{\rho} \frac{\partial}{\partial z} \left( \mu \frac{\partial u}{\partial z} \right) \]

• Frictional force per unit mass in x-direction
  \[ F_{rx} = \nu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right] \]
  where \( \nu = \mu / \rho \)
Surface wind stress:

\[ \left( \tau_{\text{wind}_x}, \tau_{\text{wind}_y} \right) = \rho_{\text{air}} c_D u_{10} (u_a, v_a) \]

\[ F_x = \frac{\text{force per unit area}}{\text{mass per unit area}} = \frac{\tau_x(z + \delta z) - \tau_x(z)}{\rho_{\text{ref}} \delta z} = \frac{1}{\rho_{\text{ref}}} \frac{\partial \tau_x}{\partial z}, \]

\[ F = \frac{1}{\rho_{\text{ref}}} \frac{\partial \tau}{\partial z} \]

(from John Marshall and R. Alan Plumb’s Atmosphere, Ocean and Climate Dynamics: An Introductory Text)
Step 1: Surface Winds

**Figure 9.1** Winds, driven by uneven solar heating and Earth's spin, drive the movement of the ocean's surface currents. The prime movers are the powerful westerlies and the persistent trade winds (easterlies).

**Figure 9.2** A combination of four forces—surface winds, the sun's heat, the Coriolis effect, and gravity—circulates the ocean surface clockwise in the Northern Hemisphere and counterclockwise in the Southern Hemisphere, forming gyres.

(Figure from *Oceanography* by Tom Garrison)
Step 2: Ekman Layer
(friictional force + Coriolis Force)

(Figure from Oceanography by Tom Garrison)
Step 3: Geostrophic Current
(Pressure Gradient Force + Coriolis Force)

NASA-TOPEX Observations of Sea-Level Height

(from Oceanography by Tom Garrison)
Step 4: Boundary Currents

(Figure from Oceanography by Tom Garrison)
History / Wind-Driven Circulation

(from Robert H. Stewart’s book on “Introduction to Physical Oceanography”)

Table 9.2 Contributions to the Theory of the Wind-Driven Circulation

<table>
<thead>
<tr>
<th>Name</th>
<th>Year</th>
<th>Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fridtjof Nansen</td>
<td>1898</td>
<td>Qualitative theory, currents transport water at an angle to the wind.</td>
</tr>
<tr>
<td>Vagn Walfrid Ekman</td>
<td>1902</td>
<td>Quantitative theory for wind-driven transport at the sea surface.</td>
</tr>
<tr>
<td>Henry Stommel</td>
<td>1948</td>
<td>Theory for westward intensification of wind-driven circulation (western boundary currents).</td>
</tr>
<tr>
<td>Walter Munk</td>
<td>1950</td>
<td>Quantitative theory for main features of the wind-driven circulation.</td>
</tr>
<tr>
<td>Kirk Bryan</td>
<td>1963</td>
<td>Numerical models of the oceanic circulation.</td>
</tr>
<tr>
<td>Bert Semtner and Robert Chervin</td>
<td>1988</td>
<td>Global, eddy-resolving, realistic model of the ocean’s circulation.</td>
</tr>
</tbody>
</table>
Step 1: Surface Winds

(Figure from *Oceanography* by Tom Garrison)
Why an Angle btw Wind and Iceberg Directions?

Figure 9.2 The balance of forces acting on an iceberg in a wind on a rotating Earth.

(from Robert H. Stewart’s book on “Introduction to Physical Oceanography”)

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Step 2: Ekman Layer
(frictional force + Coriolis Force)

(Figure from *Oceanography* by Tom Garrison)
In the Boundary Layer

For a steady state, homogeneous boundary layer

Coriolis force balances frictional force

\[ f v + A_z \frac{\partial^2 u}{\partial z^2} = 0 \]
\[ -f u + A_z \frac{\partial^2 v}{\partial z^2} = 0 \]

viscosity

\[ u = V_0 \exp(az) \cos(\pi/4 + az) \]
\[ v = V_0 \exp(az) \sin(\pi/4 + az) \]

\[ a = \sqrt{\frac{f}{2A_z}} \]
Frictional (Viscous) Force

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  \[ \mathbf{F} = \mu A u_0 \]

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  \[ F_{rx} = \nu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right] \]

where \( \nu = \frac{\mu}{\rho} \)
Surface Wind Stress

Surface wind stress:

\[
\begin{align*}
(\tau_{\text{wind}_x}, \tau_{\text{wind}_y}) &= \rho_{\text{air}} C_D u_{10} (u_a, v_a) \\
F_x &= \frac{\text{force per unit area}}{\text{mass per unit area}} = \frac{\tau_x (z + \delta z) - \tau_x (z)}{\rho_{\text{ref}} \delta z} = \frac{1}{\rho_{\text{ref}}} \frac{\partial \tau_x}{\partial z}, \\
F &= \frac{1}{\rho_{\text{ref}}} \frac{\partial \tau}{\partial z}
\end{align*}
\]

(from John Marshall and R. Alan Plumb’s Atmosphere, Ocean and Climate Dynamics: An Introductory Text)
How Deep is the Ekman Layer?

- $D \propto \left( \frac{\nu}{f} \right)^{1/2}$

$\nu = \text{vertical diffusivity of momentum}$

$f = \text{Coriolis parameter} = 2\Omega \sin \phi$

The thickness of the Ekman layer is arbitrary because the Ekman currents decrease exponentially with depth. Ekman proposed that the thickness be the depth $D_E$ at which the current velocity is opposite the velocity at the surface, which occurs at a depth $D_E = \pi/a$

$$D_E = \sqrt{\frac{2\pi^2 A_z}{f}}$$

(from Robert H. Steward)
Ekman Transport

\[
U_E = \int_{-\infty}^{0} u_E \, dz = \frac{\tau_y}{\rho_o f}; \quad V_E = \int_{-\infty}^{0} v_E \, dz = -\frac{\tau_x}{\rho_o f}
\]

\[
f u + A_z \frac{\partial^2 u}{\partial z^2} = 0
\]

\[
-f u + A_z \frac{\partial^2 v}{\partial z^2} = 0
\]

or

\[
\rho f V + \frac{\partial T_{xz}}{\partial z} = 0
\]

\[
\rho f U - \frac{\partial T_{yz}}{\partial z} = 0
\]

(Figure from The Earth System)
Ekman Transport and Ekman Pumping

(from The Earth System)

(from Talley 2011)
Ekman Transport $\rightarrow$ Convergence/Divergence

(Figure from *The Earth System*)

Surface wind + Coriolis Force

Ekman Transport

Convergence/divergence (in the center of the gyre)

Pressure Gradient Force

Geostrophic Currents

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Step 3: Geostrophic Current (Pressure Gradient Force + Coriolis Force)

NASA-TOPEX Observations of Sea-Level Height

(from Oceanography by Tom Garrison)
Geostrophic Current

Forces

Westerly winds

Eastward-flowing current

Ekman transport causes water to pile up in the gyre

Northeast trade winds

Westward-flowing current

Geostrophic Gyre Currents

Clockwise gyre resulting from geostrophic flow

(Figure from The Earth System)
Characteristics of the Gyres

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**Western boundary current (jet stream of ocean)**
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Volume transport unit:
1 sv = 1 Sverdrup = 1 million m³/sec
(the Amazon river has a transport of \(\sim 0.17 \text{ Sv}\))
Ekman Transport → Convergence/Divergence

(Figure from *The Earth System*)

Surface wind + Coriolis Force

向下

Ekman Transport

向下

Convergence/divergence (in the center of the gyre)

向下

Pressure Gradient Force

向下

Geostrophic Currents

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Theories that Explain the Wind-Driven Ocean Circulation

- Harald Sverdrup (1947) showed that the circulation in the upper kilometer or so of the ocean is directly related to the curl of the wind stress if the Coriolis force varies with latitude.
- Henry Stommel (1948) showed that the circulation in oceanic gyres is asymmetric also because the Coriolis force varies with latitude.
- Walter Munk (1950) added eddy viscosity and calculated the circulation of the upper layers of the Pacific.
- Together the three oceanographers laid the foundations for a modern theory of ocean circulation.

(from Robert H. Stewart’s book on “Introduction to Physical Oceanography”)

Sverdrup’s Theory of the Oceanic Circulation

\[ V = k \cdot \frac{\nabla \times \tau}{\beta}. \]

- The Sverdrup balance, or Sverdrup relation, is a theoretical relationship between the wind stress exerted on the surface of the open ocean and the vertically integrated meridional (north-south) transport of ocean water.

- Positive wind stress curl \( \rightarrow \) Northward mass transport

- Negative wind stress curl \( \rightarrow \) Southward mass transport

(from Talley 2011)
Global Surface Currents

(from Climate System Modeling)
Sverdrup Transport

\[
\frac{\partial p}{\partial x} = f \rho v + \frac{\partial T_{xz}}{\partial z} \quad \frac{\partial p}{\partial y} = -f \rho u + \frac{\partial T_{yz}}{\partial z}
\]

\[
\frac{\partial P}{\partial x} = \int_{-D}^{0} \frac{\partial P}{\partial x} \, dz, \quad \frac{\partial P}{\partial y} = \int_{-D}^{0} \frac{\partial P}{\partial y} \, dz,
\]

\[
M_x \equiv \int_{-D}^{0} \rho u(z) \, dz, \quad M_y \equiv \int_{-D}^{0} \rho v(z) \, dz,
\]

\[
\frac{\partial P}{\partial x} = f M_y + T_x, \quad \frac{\partial P}{\partial y} = -f M_x + T_y
\]

\[
d/dy \left( \frac{\partial P}{\partial x} = f M_y + T_x \right) - d/dx \left( \frac{\partial P}{\partial y} = -f M_x + T_y \right) \quad \text{and use} \quad \frac{\partial M_x}{\partial x} + \frac{\partial M_y}{\partial y} = 0
\]

\[
\beta M_y = \frac{\partial T_y}{\partial x} - \frac{\partial T_x}{\partial y}
\]

\[
\beta M_y = \text{curl}_z (T)
\]
Sverdrup, Geostrophic, and Ekman Transports

\[ V = \hat{k} \cdot \frac{\nabla \times \tau}{\beta}. \]

- Continuity equation for an incompressible flow:
  \[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \]
- Assume the horizontal flows are geostrophic:
  \[ \frac{\partial u_g}{\partial x} + \frac{\partial v_g}{\partial y} + \frac{\partial w}{\partial z} = 0 \]
- Replace the geostrophic flow pressure gradients:
  \[ f u_g = -\frac{1}{\rho} \frac{\partial P}{\partial y} \]
  \[ f v_g = \frac{1}{\rho} \frac{\partial P}{\partial y} \]
- The continuity equation becomes:
  \[ -\frac{\beta}{f} v_g + \frac{\partial w}{\partial z} = 0 \quad \Rightarrow \quad \beta v_g = f \frac{\partial w}{\partial z} \]

Ekman layer pumping
- vertical depth decreases
- move equatorward to conserve absolute vorticity

Ekman layer suction
- vertical depth increases
- move poleward to conserve absolute vorticity.

\[ (\zeta + f)/h = \eta/h = \text{Const} \]
Continuity equation for an incompressible flow:
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
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Assume the horizontal flows are geostrophic:
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\]

The continuity equation becomes:
\[
-\frac{\beta}{f} v_g + \frac{\partial w}{\partial z} = 0 \quad \Rightarrow \quad \beta v_g = f \frac{\partial w}{\partial z}
\]

Integrate the equation from the bottom of the upper ocean \((D_w)\) to the bottom of the Ekman layer \((D_E)\):
\[
\int_{z=-D_E}^{z=-D_W} \beta v dz = f \left[ w_E - w(-D_W) \right]
\]

Therefore, we obtain:
\[
Sverdrup transport = Geostrophic transport + Ekman transport
\]
Ekman and Sverdrup Transports

Ekman Pumping (w<0)  Ekman Suction (w>0)
Equatorward Sverdrup Transport  Poleward Sverdrup Transport

\[ V_c = \hat{k} \cdot \nabla \times \tau \]

\[ V_E = -\frac{\tau_x}{\rho_o f} \]
Ekman Pumping and Thermocline

Figure 10.1: The ocean comprises a warm, salty, stratified lens of fluid, the thermocline, circulating on top of a cold, fresh, relatively well mixed, abyss. The surface layer, above the horizontal dotted line at a depth of about 100 m, is driven directly by the wind. The thermocline below is brought into motion through a pattern of vertical velocity driven by the wind (Ekman pumping and suction) which induces flow in the ocean beneath.

(from John Marshall and R. Alan Plumb’s Atmosphere, Ocean and Climate Dynamics: An Introductory Text)
Step 4: Boundary Currents

(Figure from Oceanography by Tom Garrison)
Stommel’s Theory of Western Boundary Currents

Stommel’s Theory added bottom friction into the same equations used by Svedrup.

Surface stress:

\[ \frac{\partial p}{\partial x} = f \rho v + \frac{\partial T_{xz}}{\partial z} \]

\[ \frac{\partial p}{\partial y} = -f \rho u + \frac{\partial T_{yz}}{\partial z} \]

Bottom stress:

\[ \left( A_z \frac{\partial u}{\partial z} \right)_0 = -T_x = -E \cos(\pi y/b) \]

\[ \left( A_z \frac{\partial u}{\partial z} \right)_D = -R u \]

\[ \left( A_z \frac{\partial v}{\partial z} \right)_0 = -T_y = 0 \]

\[ \left( A_z \frac{\partial v}{\partial z} \right)_D = -R v \]

Figure 11.5 Stream function for flow in a basin as calculated by Stommel (1948). **Left:** Flow for non-rotating basin or flow for a basin with constant rotation. **Right:** Flow when rotation varies linearly with y.

(from Robert H. Stewart’s book on “Introduction to Physical Oceanography”)

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Munk’s Theory of Western Boundary Currents

Munk (1950) built upon Sverdrup’s theory, adding lateral eddy viscosity, to obtain a solution for the circulation within an ocean basin.

To simplify the equations, Munk used the mass-transport stream function $\Psi$.

(from Robert H. Stewart’s book on “Introduction to Physical Oceanography”)

\[ \frac{1}{\rho} \frac{\partial p}{\partial x} = f v + \frac{\partial}{\partial z} \left( A_s \frac{\partial u}{\partial z} \right) + A_H \frac{\partial^2 u}{\partial x^2} + A_H \frac{\partial^2 u}{\partial y^2} \]

\[ \frac{1}{\rho} \frac{\partial p}{\partial y} = -f u + \frac{\partial}{\partial z} \left( A_s \frac{\partial v}{\partial z} \right) + A_H \frac{\partial^2 v}{\partial x^2} + A_H \frac{\partial^2 v}{\partial y^2} \]

mass-transport stream function $\Psi$

\[ A_H \nabla^4 \Psi - \beta \frac{\partial \Psi}{\partial x} = -\text{curl}_z T \]

Friction \hspace{0.2cm} Sverdrup Balance

Friction

Sverdrup Balance

\[ M_x \equiv \frac{\partial \Psi}{\partial y}, \quad M_y \equiv -\frac{\partial \Psi}{\partial x} \]
Why Strong Boundary Currents?
A Potential Vorticity View

(Figure from *The Earth System*)

**Goal:** Maintain the “steady state” of the negative vorticity induced by wind stress curve

\[ \xi^- = \xi^- + \xi^+ \]

friction has to be big \(\Rightarrow\) strong boundary current
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Volume transport unit:
- \(1 \text{ sv} = 1 \text{ Sverdrup} = 1 \text{ million } m^3/\text{sec}\)
- (the Amazon river has a transport of ~0.17 Sv)
Gulf Stream

- Speed = 2 m/sec
- Depth = 450 m
- Width = 70 Km
- Color: clear and blue

(Figure from Oceanography by Tom Garrison)