Lecture 10: Ocean Circulation



Wind-Driven Circulation
 Ekman Layer, Transport, Pumping
 Sverdrup Theory
 Western Boundary Current



Basic Ocean Current Systems



Thermohaline Conveyor Belt



(Figure from Climate System Modeling)



Global Surface Currents



(from *Climate System Modeling*)



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Six Great Current Circuits in the World Ocean





 5 of them are geostrophic gyres: North Pacific Gyre
 South Pacific Gyre
 North Atlantic Gyre
 South Atlantic Gyre
 Indian Ocean Gyre

The 6th and the largest current:
 Antarctic Circumpolr Current
 (also called West Wind Drift)

(Figure from *Oceanography* by Tom Garrison)



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Characteristics of the Gyres

(Figure from Oceanography by Tom Garrison)



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Trade wind-driven current

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Global Surface Currents



(from *Climate System Modeling*)



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Basic Ocean Current Systems





Frictional (Viscous) Force



- Any real fluid is subject to internal friction (viscosity)
- Force required to maintain this flow
 - $\rightarrow F = \mu A u_0 / l$
- For a layer of fluid at depth δZ , the force is $\Rightarrow F = \mu A \delta u / \delta z$
- Viscous force per unit area (shearing stress):

$$\Rightarrow \tau_{zx} = \lim_{\delta z \to 0} \mu \frac{\delta u}{\delta z} = \mu \frac{\partial u}{\partial z}$$



Stresses applied on a fluid element

$$\Rightarrow \left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} \frac{\delta z}{2}\right) \delta x \, \delta y - \left(\tau_{zx} - \frac{\partial \tau_{zx}}{\partial z} \frac{\delta z}{2}\right) \delta x \, \delta y$$

• Viscous force per unit mass due to stress

$$\Rightarrow \ \frac{1}{\rho} \frac{\partial \tau_{zx}}{\partial z} = \frac{1}{\rho} \frac{\partial}{\partial z} \left(\mu \frac{\partial u}{\partial z} \right)$$

• Frictional force per unit mass in x-direction

$$\Rightarrow F_{rx} = v \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right]$$

where $v = \mu/\rho$
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Surface Wind Stress



Step 1: Surface Winds



Figure 9.1 Winds, driven by uneven solar heating and Earth's spin, drive the movement of the ocean's surface currents. The prime movers are the powerful westerlies and the persistent trade winds (easterlies).



Figure 9.2 A combination of four forces-surface winds, the sun's heat, the Coriolis effect, and gravity-circulates the ocean surface clockwise in the Northern Hemisphere and counterclockwise in the Southern Hemisphere, forming gyres.

(Figure from *Oceanography* by Tom Garrison)



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Step 2: Ekman Layer (frictional force + Coriolis Force)



(Figure from Oceanography by Tom Garrison)



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Step 3: Geostrophic Current (Pressure Gradient Force + Corioils Foce)



NASA-TOPEX Observations of Sea-Level Hight



(from *Oceanography* by Tom Garrison)



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Step 4: Boundary Currents



(Figure from Oceanography by Tom Garrison)



History / Wind-Driven Circulation

(from Robert H. Stewart's book on "Introduction to Physical Oceanography")

	Table 9.2 Contributions to the Theory of the Wind-Driven Circulation		
	Fridtjof Nansen	(1898)	Qualitative theory, currents transport water at an angle to the wind.
	Vagn Walfrid Ekman	(1902)	Quantitative theory for wind-driven transport at the sea surface
	-Harald Sverdrup	(1947)	Theory for wind-driven circulation in the eastern Pacific.
	Henry Stommel	(1948)	Theory for westward intensification of wind-driven circulation (western boundary currents).
	Walter Munk	(1950)	Quantitative theory for main features of the wind- driven circulation.
	Kirk Bryan	(1963)	Numerical models of the oceanic circulation.
	Bert Semtner	(1988)	Global, eddy-resolving, realistic model of the
	and Robert Chervin		ocean's circulation.
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(Figure from *Oceanography* by Tom Garrison)



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Why an Angle btw Wind and Iceberg Directions?



Step 2: Ekman Layer (frictional force + Coriolis Force)



(Figure from Oceanography by Tom Garrison)



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In the Boundary Layer

For a steady state, homogeneous boundary layer

 $fv + A_z \frac{\partial^2 u}{\partial z^2} = 0$ $-fu + A_z \frac{\partial^2 v}{\partial z^2} = 0$

Coriolis force balances frictional force

viscosity

 $u = V_0 \exp(az) \cos(\pi/4 + az)$ $v = V_0 \exp(az) \sin(\pi/4 + az)$

 $a = \sqrt{\frac{f}{2A_z}}$





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Surface Wind Stress



How Deep is the Ekman Layer?



Fig. 4.4 (a) Vertical distribution of temperature and salinity at 50°N., 145°W. in early September, 1977. The solid lines are before a storm and the dotted lines are after a storm, which depict the vertical mixing above the seasonal thermocline. The main thermocline, or pycnocline in this area is between 110 m and 160 m depth.
(b) Time-averaged velocity for a 25 day summer period at an open ocean site southwest of Bermuda. Current meter measured velocity is referenced to 70 m. The topmost dashed vector is the time-averaged wind stress (Price et al., 1986).

 \Box D \propto (v/f)^{1/2}

v = vertical diffusivity of momentum f = Coriolis parameter = $2\Omega \sin\phi$

The thickness of the Ekman layer is arbitrary because the Ekman currents decrease exponentially with depth. Ekman proposed that the thickness be the depth D_E at which the current velocity is opposite the velocity at the surface, which occurs at a

depth
$$D_E = \pi/a$$

$$D_E = \sqrt{\frac{2\pi^2 A_z}{f}}$$

(from *Robert H. Steward*)



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Ekman Transport



Ekman Transport and Ekman Pumping





Ekman Transport \rightarrow Convergence/Divergence

(Figure from *The Earth System*)



Step 3: Geostrophic Current (Pressure Gradient Force + Corioils Foce)



NASA-TOPEX Observations of Sea-Level Hight



(from *Oceanography* by Tom Garrison)



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Geostrophic Current



(Figure from *The Earth System*)



Characteristics of the Gyres

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Ekman Transport \rightarrow Convergence/Divergence

(Figure from *The Earth System*)



Theories that Explain the Wind-Driven Ocean Circulation

- Harald Sverdrup (1947) showed that the circulation in the upper kilometer or so of the ocean is directly related to the curl of the wind stress if the Coriolis force varies with latitude.
- Henry Stommel (1948) showed that the circulation in oceanic gyres is asymmetric also because the Coriolis force varies with latitude.
- Walter Munk (1950) added eddy viscosity and calculated the circulation of the upper layers of the Pacific.
- Together the three oceanographers laid the foundations for a modern theory of ocean circulation.

(from Robert H. Stewart's book on "Introduction to Physical Oceanography")

Sverdrup's Theory of the Oceanic Circulation

$$V = \hat{\mathbf{k}} \cdot \frac{\nabla \times \tau}{\beta}.$$

The Sverdrup balance, or Sverdrup relation, is a theoretical relationship between the wind stress exerted on the surface of the open ocean and the vertically integrated meridional (north-south) transport of ocean water.



Positive wind stress curl
 Northward mas transport
 Negative wind stress curl
 Southward mass transport

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Global Surface Currents



(from *Climate System Modeling*)



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Sverdrup Transport

$$\frac{\partial p}{\partial x} = f \rho v + \frac{\partial T_{xz}}{\partial z} \qquad \frac{\partial p}{\partial y} = -f \rho u + \frac{\partial T_{yz}}{\partial z}$$

$$\frac{\partial P}{\partial x} = \int_{-D}^{0} \frac{\partial p}{\partial x} dz, \qquad \frac{\partial P}{\partial y} = \int_{-D}^{0} \frac{\partial p}{\partial y} dz,$$

$$M_x = \int_{-D}^{0} \rho u(z) dz, \qquad M_y = \int_{-D}^{0} \rho v(z) dz,$$

$$\frac{\partial P}{\partial x} = f M_y + T_x$$

$$\frac{\partial P}{\partial y} = -f M_x + T_y$$

$$d/dy \left(\frac{\partial P}{\partial x} = f M_y + T_x \right) - d/dx \left(\frac{\partial P}{\partial y} = -f M_x + T_y \right) \text{ and use } \frac{\partial M_x}{\partial x} + \frac{\partial M_y}{\partial y} = 0$$

$$\beta M_y = \frac{\partial T_y}{\partial x} - \frac{\partial T_x}{\partial y}$$

$$\beta M_y = \operatorname{curl}_z(T)$$

Sverdrup, Geostrophic, and Ekman Transports

$$V = \hat{\mathbf{k}} \cdot \frac{\nabla \times \tau}{\beta}.$$

• Continuity equation for an incompressible flow:

 $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

• Assume the horizontal flows are geostrophic:

$$\frac{\partial u_{g}}{\partial x} + \frac{\partial v_{g}}{\partial x} + \frac{\partial w}{\partial z} = 0$$

• Replace the geostrophic flow pressure gradients:

$$fu_{g} = -\frac{1}{\rho} \frac{\partial P}{\partial y}$$
$$fv_{g} = \frac{1}{\rho} \frac{\partial P}{\partial y}$$

• The continuity equation becomes:

$$\frac{-\beta}{f}v_{g} + \frac{\partial w}{\partial z} = 0 \quad \Longrightarrow \quad \beta v_{g} = f \frac{\partial w}{\partial z}$$



Ekman layer pumping

 \rightarrow vertical depth decreases

→ move equatorward to conserve absolute vorticity.



Ekman layer suction

- \rightarrow vertical depth increases
- \rightarrow move poleward to conserve absolute vorticity.

$$(\zeta + f)/h = \eta/h = \text{Const}$$

Sverdrup, Geostrophic, and Ekman Transports

$$V = \hat{\mathbf{k}} \cdot \frac{\nabla \times \tau}{\beta}$$
. $V_E = \int_{-\infty}^{0} v_E \, dz = -\frac{\tau_x}{\rho_o f}$

• Continuity equation for an incompressible flow:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

• Assume the horizontal flows are geostrophic:

$$\frac{\partial u_s}{\partial x} + \frac{\partial v_s}{\partial x} + \frac{\partial w}{\partial z} = 0$$

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• The continuity equation becomes:

$$\frac{-\beta}{f}v_{g} + \frac{\partial w}{\partial z} = 0 \quad \Longrightarrow \quad \beta v_{g} = f\frac{\partial w}{\partial z}$$

• Integrate the equation from the bottom of the upper ocean (D_w) to the bottom of the Ekman layer (D_E):

 $U_E = \int_{-\infty}^{0} u_E \, dz = \frac{\tau_y}{\rho_o f}; \qquad V_E = \int_{-\infty}^{0} \upsilon_E \, dz = -\frac{\tau_x}{\rho_o f}$

$$\beta \int_{z=-D_{w}}^{z=-D_{\varepsilon}} v \partial z = f \left[w_{E} - w(-D_{w}) \right]^{\text{assume zero}}$$

• Ekman pumping (W_E) is related to the convergence of the Ekman transport:

$$w(-D_E) = \frac{\partial}{\partial x} \left(\frac{\tau^y}{\rho f} \right) - \frac{\partial}{\partial y} \left(\frac{\tau^x}{\rho f} \right)$$

• Therefore, we obtain:

$$\int_{z=-D_w}^{z=-D_e} v \partial z = \frac{1}{\rho \beta} \left(\frac{\partial \tau_w^y}{\partial x} - \frac{\partial \tau_w^x}{\partial y} \right) + \frac{1}{\rho f} \tau_w^x$$
geostrophic Sverdrup - (Ekma transport Transport

• Therefore,

Sverdrup transport = Geostrophic transport + Ekman transport



Ekman Pumping and Thermocline



Figure 10.1: The ocean comprises a warm, salty, stratified lens of fluid, the thermocline, circulating on top of a cold, fresh, relatively well mixed, abyss. The surface layer, above the horizontal dotted line at a depth of about 100 m, is driven directly by the wind. The thermocline below is brought in to motion through a pattern of vertical velocity driven by the wind (Ekman pumping and suction) which induces flow in the ocean beneath.

(from John Marshall and R. Alan Plumb's Atmosphere, Ocean and Climate Dynamics: An Introductory Text)

Step 4: Boundary Currents



(Figure from Oceanography by Tom Garrison)



Stommel's Theory of Western Boundary Currents



Figure 11.5 Stream function for flow in a basin as calculated by Stommel (1948). Left: Flow for non-rotating basin or flow for a basin with constant rotation. Right: Flow when rotation varies linearly with y.

Stommel's Theory added bottom friction into the same equations used by Svedrup.

$$\frac{\partial p}{\partial x} = f \rho v + \frac{\partial T_{xz}}{\partial z} \qquad \frac{\partial p}{\partial y} = -f \rho u + \frac{\partial T_{yz}}{\partial z}$$

$$\left(A_z \frac{\partial u}{\partial z}\right)_0^{-} = -T_x = -F \cos(\pi y/b) \qquad \left(A_z \frac{\partial u}{\partial z}\right)_D^{-} = -R u$$

$$\left(A_z \frac{\partial v}{\partial z}\right)_0^{-} = -T_y = 0 \qquad \left(A_z \frac{\partial v}{\partial z}\right)_D^{-} = -R v$$
surface stress bottom stress ESS228

(from Robert H. Stewart's book on "Introduction to Physical Oceanography")

Munk's Theory of Western Boundary Currents



- Munk (1950) built upon Sverdrup's theory, adding lateral eddy viscosity, to obtain a solution for the circulation within an ocean basin.
- To simplify the equations, Munk used the mass-transport stream function. (from Robert H. Stewart's book on "*Introduction to Physical Oceanography*")

Why Strong Boundary Currents? A Potential Vorticity View



Goal: Maintain the "steady state" of the negative vorticity induced by wind stress curve

$$\xi - = \xi - \text{plus} \xi + \xi - = \xi + \text{plus} \xi +$$

strong boundary

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Gulf Stream



Warm

water

d

Warm

water



(Figure from Oceanography by Tom Garrison)



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