

Geophysics Fluid Dynamics (ESS228)

- **Course Time**
Lectures: Tu, Th 09:30-10:50
Discussion: 3315 Croul Hall
- **Text Book**
J. R. Holton, "*An introduction to Dynamic Meteorology*", Academic Press (Ch. 1, 2, 3, 4, 6, 8, 11).
Adrian E. Gill, "*Atmosphere-Ocean Dynamics*", Academic Press (Ch. 5, 6, 7, 8, 9, 10, 11, 12).
- **Grade**
Homework (30%), Midterm (35%), Final (35%)
- **Homework**
Issued and due every Thursday



Syllabus

SYLLABUS		
Week 1	1/10 & 1/12	Introduction and Review of Mathematical Tools Mathematical tools and estimating with scale Fundamental and apparent forces
Week 2	1/17 & 1/19	Basic Conservation Laws Equations of motion, thermodynamic energy equation, Continuity equation
Week 3	1/24 & 1/26	Applications of the Equations of Motion Balanced (geostrophic, inertial, cyclostrophic, gradient) flows Thermal wind balance
Week 4	1/31 & 2/2	Circulation, Vorticity, and Divergence The Circulation theorem Vorticity and potential vorticity
Week 5	2/7 & 2/9	Waves in the Atmosphere Perturbation method Gravity wave, Rossby wave, Kelvin wave
Midterm	2/14	
Week 6	2/16	Adjustment Under Gravity In a non-rotating system
Week 7	2/21 & 2/23	Adjustment Under Gravity In a density-stratified fluid Effect of rotation
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Week 9	3/6 & 3/8	Ocean Circulation Wind-driven circulation Western Boundary currents
Week 10	3/13 & 3/15	Tropical Dynamics Scale analysis of large-scale tropical motions Equatorial wave theory
Final	TBD	



Dynamics and Kinematics

- **Kinematics**: The term **kinematics** means *motion*. Kinematics is the study of motion without regard for the cause.
- **Dynamics**: On the other hand, **dynamics** is the study of the *causes* of motion.

This course discusses the physical laws that govern atmosphere/ocean motions.



Geophysical Fluid Dynamics

↓
 the study of the *causes* of motion
 ↓
 causes: solar radiation competes with gravity
 ↓
 distributions of T and motions
 ↓
 governed by conservation laws



Newton's 2nd Law of Motion

$$\frac{D_a \mathbf{U}_a}{Dt} = \sum \mathbf{F}$$

← Newton's 2nd Law of Momentum

\mathbf{U}_a = absolute velocity viewed in an inertial system

$\frac{D_a \mathbf{U}_a}{Dt}$ = rate of change of \mathbf{U}_a following the motion in an inertial system

- The conservation law for momentum (Newton's second law of motion) relates the rate of change of the absolute momentum following the motion in an inertial reference frame to the sum of the forces acting on the fluid.



Conservation of Mass

- The mathematical relationship that expresses conservation of mass for a fluid is called the *continuity equation*.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0$$

(mass divergence form)

$$\frac{1}{\rho} \frac{D\rho}{Dt} + \nabla \cdot \mathbf{U} = 0$$

(velocity divergence form)



The First Law of Thermodynamics

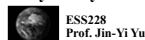
- This law states that (1) heat is a form of energy that (2) its conversion into other forms of energy is such that total energy is conserved.
- The change in the internal energy of a system is equal to the heat added to the system minus the work down by the system:

$$\Delta U = Q - W$$

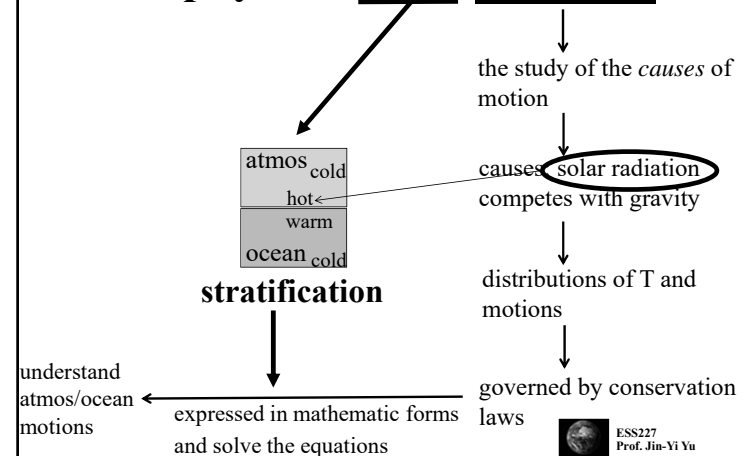
change in internal energy
(related to temperature)

Heat added to the system

Work done by the system



Geophysical Fluid Dynamics



Stratification

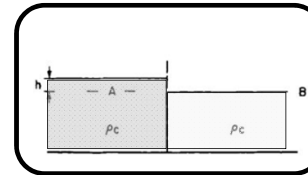
- ❑ Fluid that is less dense than its surroundings tends to rise until it has the same density as its surroundings. It will tend to become stratified.



Adjustment Under Gravity in a Non-Rotating System

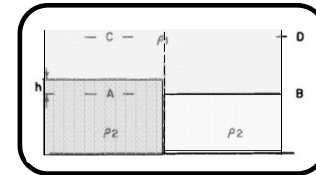
External Gravity Waves

adjustment of a homogeneous fluid with a free surface

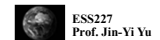
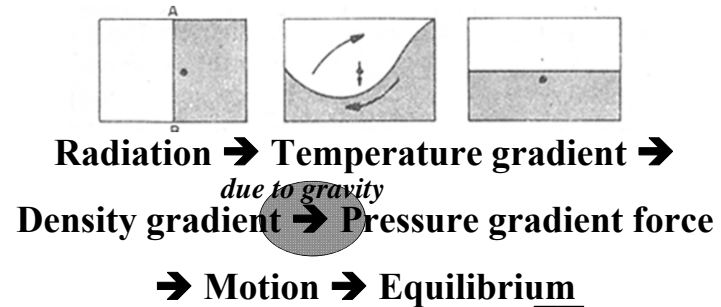
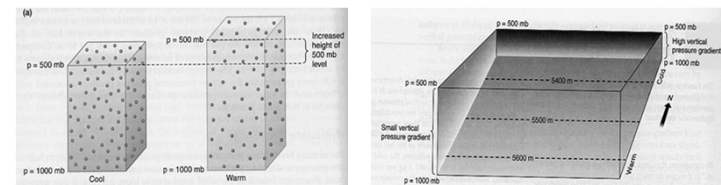
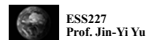
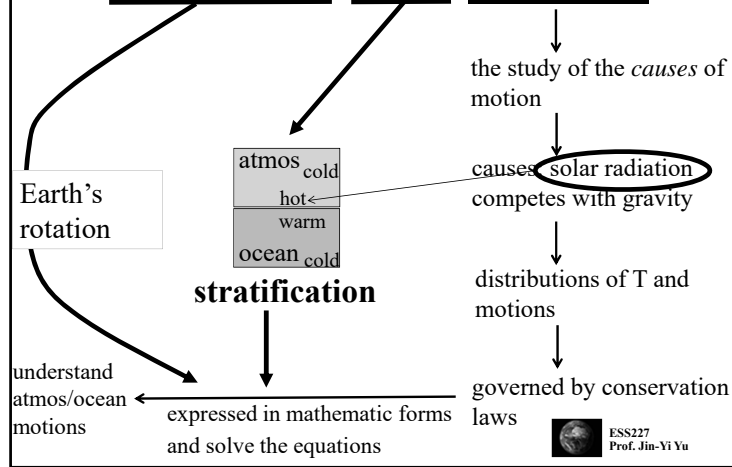


Internal Gravity Waves

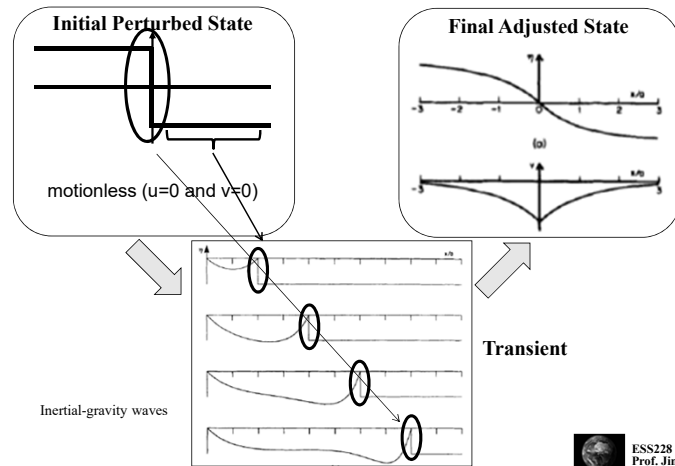
adjustment of a density-stratified fluid



Geophysical Fluid Dynamics



An Example of Geostrophic Adjustment



What Are the Issues?

- ❑ The fundamental aim is to understand the circulations of the atmosphere and ocean and the observed distributions of physical quantities such as temperature.
- ❑ The temperature distribution can be viewed as the result of a "competition" between the *sun*, which tries to warm the tropics more than the poles (and so create horizontal contrasts), and **gravity**, which tries to remove horizontal contrasts and arrange for warmer fluid to overlies colder fluid.
- ❑ This "competition" is complicated by such *effects as the rotation of the earth*, the variation of the angle between gravity and the rotation axis (the beta effect), and *contrasts between the properties of air and water*.

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Atmosphere-Ocean Dynamics

- Chapter Three Properties of a Fluid at Rest
- Chapter Four Equations Satisfied by a Moving Fluid
- Chapter Five Adjustment under Gravity in a Nonrotating System
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- Chapter Seven Effects of Rotation
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Dynamic Meteorology

- Chapter 2 Basic Conservation Laws
- Chapter 3 Elementary Applications of the Basic Equations
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Atmosphere-Ocean Dynamics

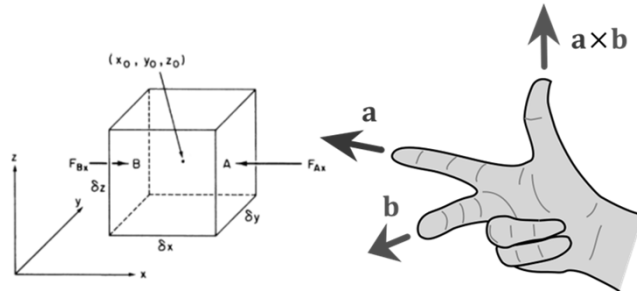
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Lecture 1: Introduction and Review



- Review of fundamental mathematical tools
- Fundamental and apparent forces



Basic Conservation Laws

Atmospheric motions are governed by three fundamental physical principles:

- conservation of mass (continuity equation)
- conservation of momentum (Newton's 2nd law of motion)
- conservation of energy (1st law of thermodynamics)

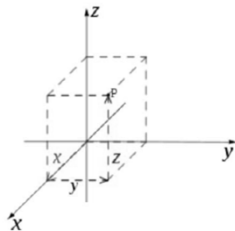
We need to develop mathematical formulas to describe these basic laws.



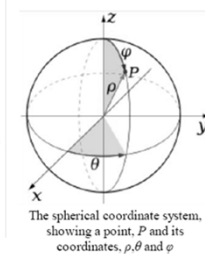
Coordinate System

A coordinate system is needed to describe the location in space.

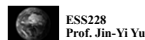
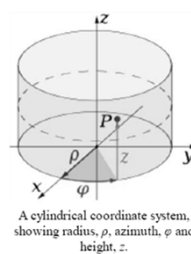
(1) Cartesian (x, y, z)



(2) Spherical (ρ, φ, θ)



(3) Cylindrical (ρ, φ, z)



Basic question

N-S equation:

$$\begin{aligned}
 & \rho \left[\frac{du}{dt} - \frac{uv \tan \phi}{r} + \frac{uw}{r} \right] + 2\Omega \rho (w \cos \phi - v \sin \phi) \\
 &= -\frac{1}{r \cos \phi} \frac{\partial p}{\partial \lambda} + (\nabla \cdot \mathbf{T}) \cdot \hat{\lambda} \\
 & \rho \left[\frac{dv}{dt} - \frac{u^2 \tan \phi}{r} + \frac{vw}{r} \right] + 2\Omega \rho u \sin \phi \\
 &= -\frac{1}{r} \frac{\partial p}{\partial \phi} + (\nabla \cdot \mathbf{T}) \cdot \hat{\phi} \\
 & \rho \left[\frac{dw}{dt} - \frac{u^2 + v^2}{r} \right] - 2\Omega \rho u \cos \phi \\
 &= -\frac{\partial p}{\partial r} - \rho g + (\nabla \cdot \mathbf{T}) \cdot \hat{r}
 \end{aligned}
 \quad \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \text{Momentum}$$

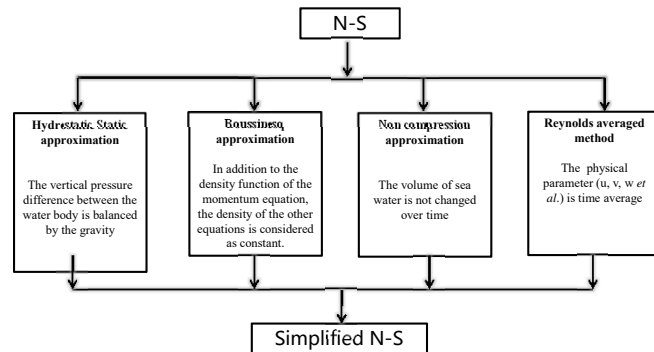
$$\frac{dp}{dt} + \frac{\rho}{r \cos \phi} \left(\frac{\partial u}{\partial \lambda} + \frac{\partial(v \cos \phi)}{\partial \phi} \right) + \frac{\rho}{r^2} \frac{\partial(r^2 w)}{\partial r} = 0 \quad \text{Continuous Thermal State}$$

$$\frac{dT}{dt} - \frac{\beta T}{\rho c_p} \frac{dp}{dt} = \frac{\nabla \cdot (\kappa \nabla T)}{\rho c_p} - \frac{\sigma}{\rho c_p}$$

$$\rho = \rho(p, T)$$

Solving N-S equations is the basic problem in the numerical model of ocean circulation

Simplification of N-S Equations



Atmosphere-Ocean Dynamics

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Waves in the Atmosphere and Oceans

Restoring Force

- Conservation of potential temperature in the presence of positive static stability → internal gravity waves
- Conservation of potential vorticity in the presence of a mean gradient of potential vorticity → Rossby waves

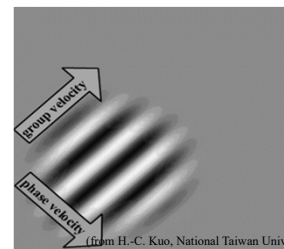
- **External gravity wave** (Shallow-water gravity wave)
- **Internal gravity (buoyancy) wave**
- **Inertial-gravity wave**: Gravity waves that have a large enough wavelength to be affected by the earth's rotation.
- **Rossby Wave**: Wavy motions results from the conservation of potential vorticity.
- **Kelvin wave**: It is a wave in the ocean or atmosphere that balances the Coriolis force against a topographic boundary such as a coastline, or a waveguide such as the equator. Kelvin wave is non-dispersive.

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Dispersion of Internal Gravity Waves

$$\hat{v} \equiv v - \bar{u}k = \pm Nk / (k^2 + m^2)^{1/2} = \pm Nk / |\kappa|$$

mean flow zonal wavenumber vertical wavenumber total wavenumber



- In the atmosphere, internal gravity waves generated in the troposphere by cumulus convection, by flow over topography, and by other processes may propagate upward many scale heights into the middle atmosphere.

\hat{v} is always smaller than N !!

Internal gravity waves can have any frequency between zero and a maximum value of N .

- Phase velocity:

$$c_x = \hat{v}/k \text{ and } c_z = \hat{v}/m$$

- Group velocity:

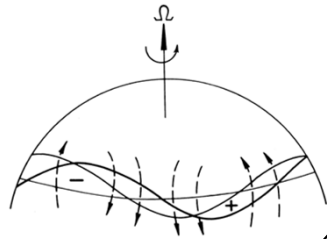
$$c_{gx} = \frac{\partial v}{\partial k} = \bar{u} \pm \frac{Nm^2}{(k^2 + m^2)^{3/2}}$$

$$c_{gz} = \frac{\partial v}{\partial m} = \pm \frac{(-Nkm)}{(k^2 + m^2)^{3/2}}$$

- Internal gravity waves thus have the remarkable property that group velocity is perpendicular to the direction of phase propagation.

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Rossby Wave



- The wave type that is of most importance for large-scale meteorological processes is the Rossby wave, or planetary wave.
- In an inviscid barotropic fluid of **constant depth** (where the divergence of the horizontal velocity must vanish), the Rossby wave is an absolute vorticity-conserving motion that owes its existence to the **variation of the Coriolis parameter** with latitude, the so-called β -effect.

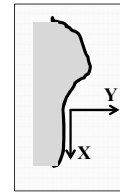
$$(\zeta + f)/h = \eta/h = \text{Const}$$

$$P \equiv (\zeta + f) \left(-g \frac{\partial \theta}{\partial p} \right) = \text{Const}$$

- More generally, in a baroclinic atmosphere, the Rossby wave is a potential vorticity-conserving motion that owes its existence to the **isentropic gradient of potential vorticity**.

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Kelvin Waves



Governing Equations

$$\frac{du}{dt} - fv = -g \frac{\partial h}{\partial x};$$

$$\frac{dv}{dt} + fu = -g \frac{\partial h}{\partial y};$$

$$\frac{dh}{dt} + D \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0.$$

A unique boundary condition
 $y = 0$ is $v = 0$

$$\begin{pmatrix} u' \\ h' \end{pmatrix} = \text{Re} \left\{ \begin{pmatrix} U(y) \\ H(y) \end{pmatrix} \exp[ik(x - ct)] \right\}$$

$$H = \text{const} \times \exp \left(-\frac{f}{c} y \right)$$

$$-\frac{f}{g} U = -\frac{f}{c} H$$

$$c = \sqrt{gD}$$

- A Kelvin wave is a type of low-frequency gravity wave in the ocean or atmosphere that balances the Earth's Coriolis force against a topographic boundary such as a coastline, or a waveguide such as the equator.
- Therefore, there are two types of Kelvin waves: coastal and equatorial.
- A feature of a Kelvin wave is that it is non-dispersive, i.e., the phase speed of the wave crests is equal to the group speed of the wave energy for all frequencies.

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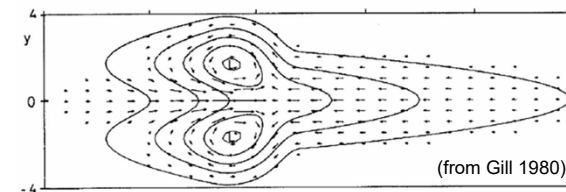
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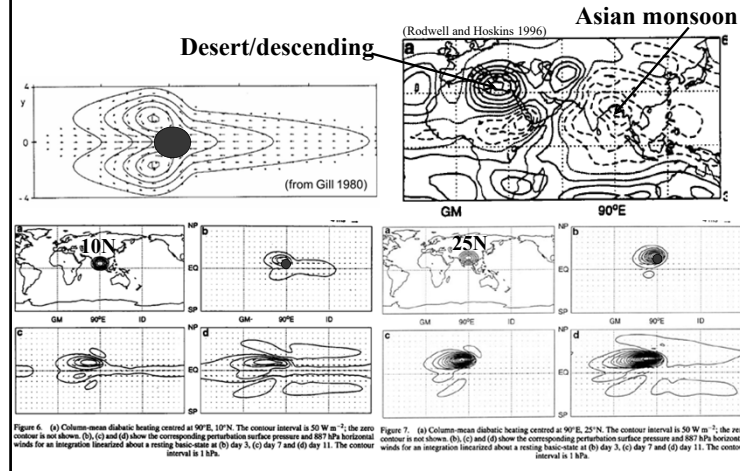
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Gill's Response to Symmetric Heating

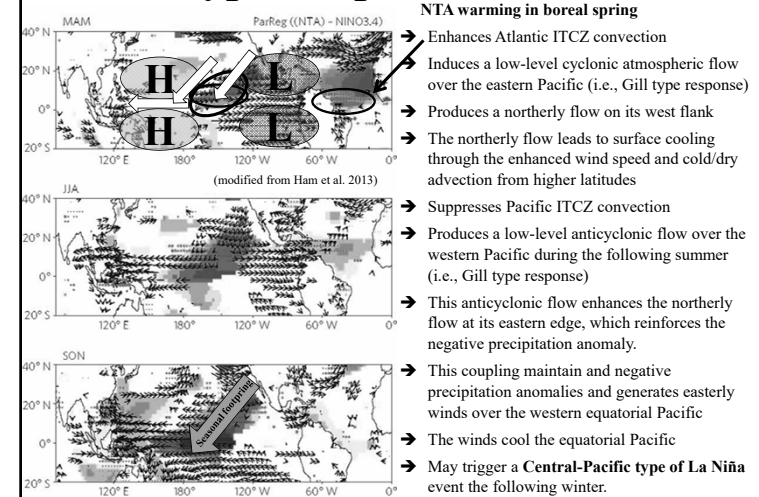


- This response consists of an eastward-propagating Kelvin wave to the east of the symmetric heating and a westward-propagating Rossby wave of $n=1$ to the west.
- The Kelvin wave low-level easterlies to the east of the heating, while the Rossby wave produces low-level westerlies to the west.
- The easterlies are trapped to the equator due to the property of the Kelvin wave.
- The $n=1$ Rossby wave consists of two cyclones symmetric and straddling the equator.
- The meridional scale of this response is controlled by the equatorial Rossby radius, which is related to the β -effect and the stability and is typically of the order of 1000km.

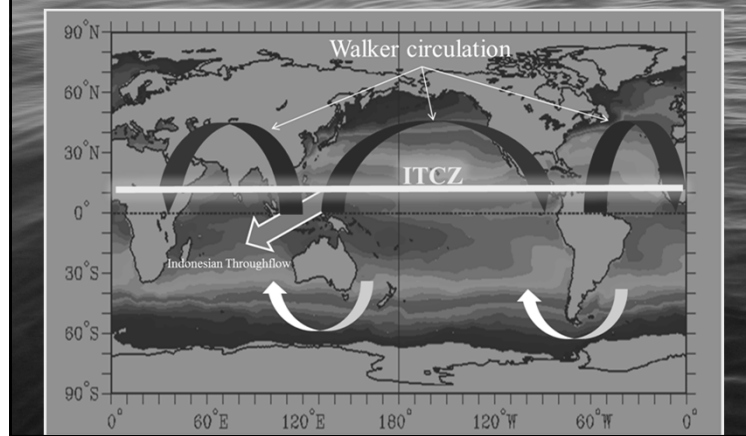
Gill Type Response to Tropical Heating



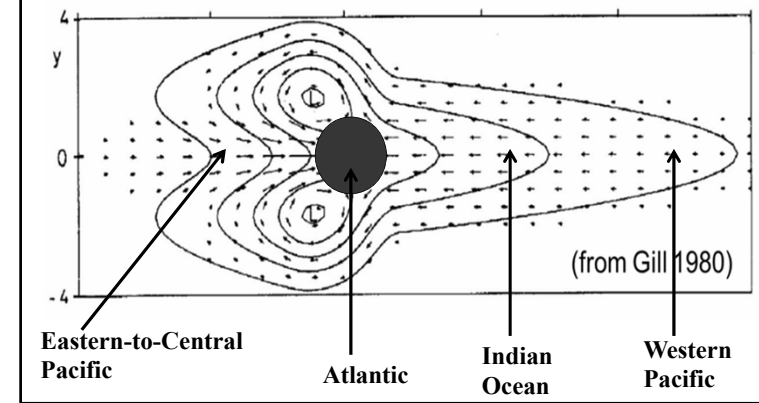
Gill-Type Response Mechanism



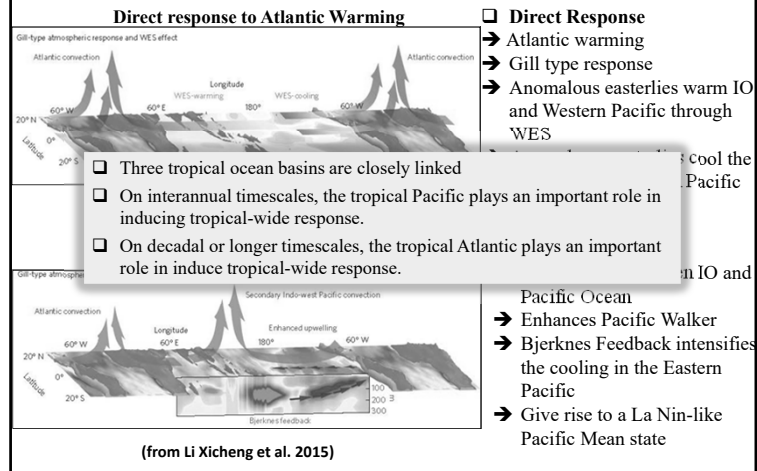
Inter-Basin Interactions (On Decadal or Longer Timescales)



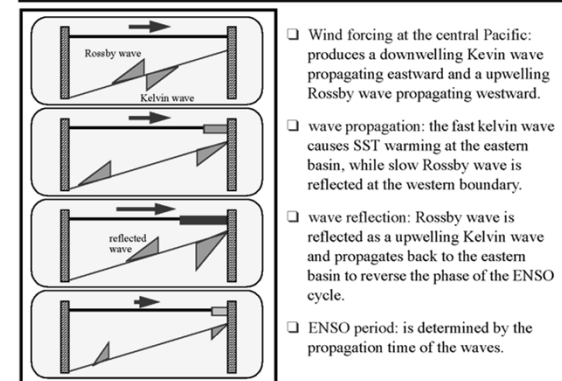
Gill Type Response to Tropical Heating



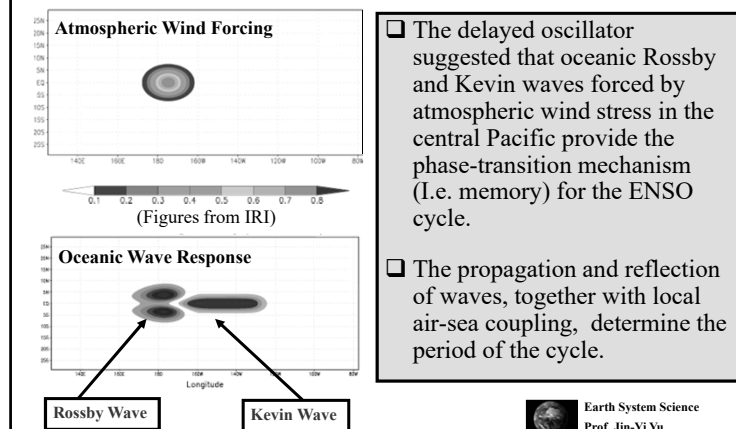
Trend: Atlantic Warming → Pacific Cooling



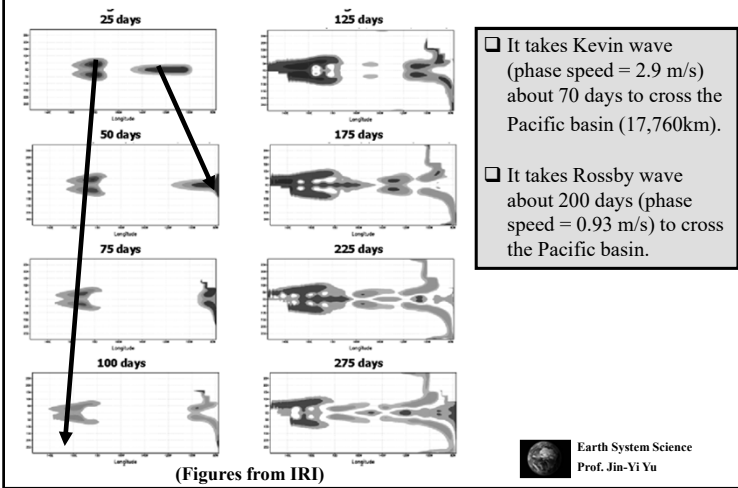
Delayed Oscillator Theory



Delayed Oscillator: Wind Forcing



Wave Propagation and Reflection



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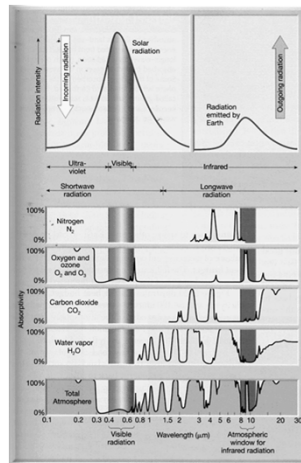
Chapter One How the Ocean-Atmosphere System Is Driven

- 1.1 Introduction
- 1.2 The Amount of Energy Received by the Earth
- 1.3 Radiative Equilibrium Models
- 1.4 The Greenhouse Effect
- 1.5 Effects of Convection
- 1.6 Effects of Horizontal Gradients
- 1.7 Variability in Radiative Driving of the Earth



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Selective Absorption and Emission



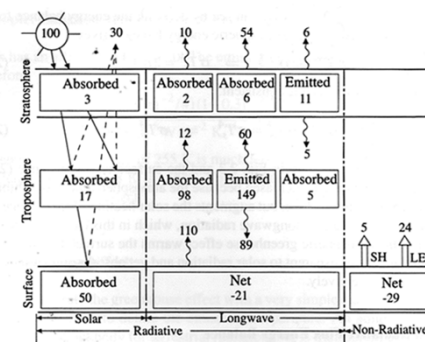
(from The Atmosphere)



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- ❑ The atmosphere is not a perfect blackbody, it absorbs some wavelength of radiation and is transparent to others (such as solar radiation). → Greenhouse effect.
- ❑ Objective that selectively absorbs radiation usually selectively emit radiation at the same wavelength.
- ❑ For example, water vapor and CO₂ are strong absorbers of infrared radiation and poor absorbers of visible solar radiation.

Vertical Distribution of Energy



(from Global Physical Climatology)

Incoming solar radiation

- 70% absorbed by Earth
- 50% by Earth's surface
- 20% by atmosphere

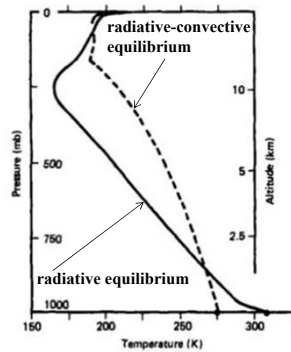
Outgoing terrestrial radiation

- 70 (units) back to space
- 21% by surface
- 49% by the atmosphere



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Effect of Convection



- ❑ **Radiative Equilibrium:** The temperature distribution that would be obtained based on the radiative energy balance in the absence of fluid motion.
- ❑ **Radiative-Convective Equilibrium:** The temperature distribution that would be obtained based on a balance between radiative and convective effects.
- ❑ Whether or not convection will occur depends on the "lapse" rate, i.e., the rate at which the temperature of the atmosphere decreases with height. Convection will only occur when the lapse rate exceeds a certain value.



Potential Temperature (θ)

- ❑ The potential temperature of an air parcel is defined as the temperature the parcel would have if it were moved adiabatically from its existing pressure and temperature to a standard pressure P_0 (generally taken as 1000mb).

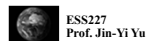
$$\theta = T \left(\frac{P_0}{P} \right)^{\frac{R}{C_p}}$$

θ = potential temperature
 T = original temperature
 P = original pressure
 P_0 = standard pressure = 1000 mb
 R = gas constant = $R_d = 287 \text{ J deg}^{-1} \text{ kg}^{-1}$
 C_p = specific heat = $1004 \text{ J deg}^{-1} \text{ kg}^{-1}$
 $R/C_p = 0.286$

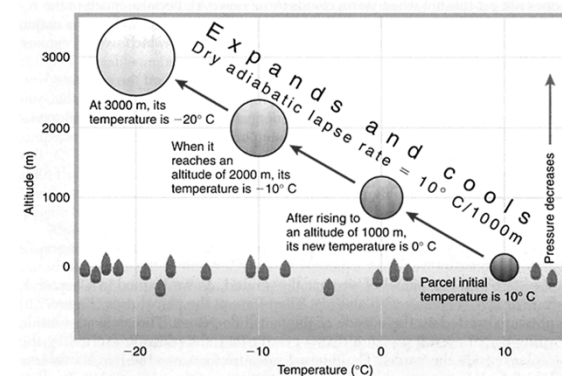


Dry and Moist Adiabatic Lapse Rates

- ❑ Dry adiabatic lapse rate is constant = $10^\circ\text{C}/\text{km}$.
 - ❑ Moist adiabatic lapse rate is NOT a constant. It depends on the temperature of saturated air parcel.
 - ❑ The higher the air temperature, the smaller the moist adiabatic lapse rate.
- ➔ When warm, saturated air cools, it causes more condensation (and more latent heat release) than for cold, saturated air.



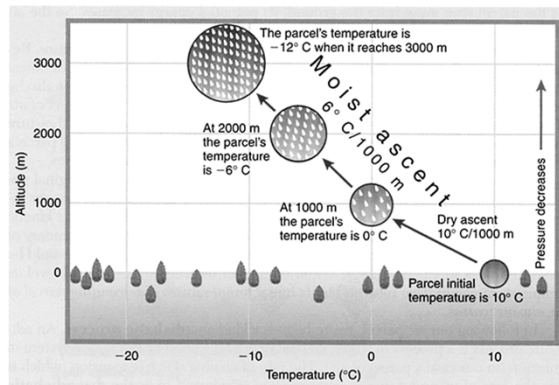
Dry Adiabatic Lapse Rate



(from *Meteorology: Understanding the Atmosphere*)



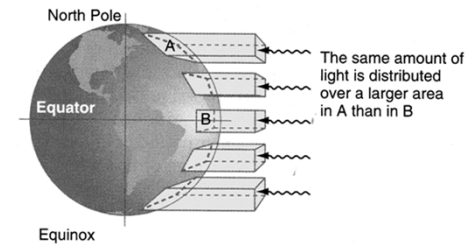
Moist Adiabatic Lapse Rate



(from *Meteorology: Understanding the Atmosphere*)

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Zenith Angle and Insolation

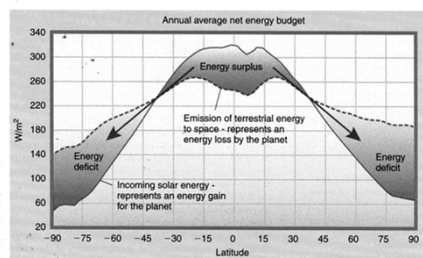


(from *Meteorology: Understanding the Atmosphere*)

- ❑ The larger the solar zenith angle, the weaker the insolation, because the same amount of sunlight has to be spread over a larger area.

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Latitudinal Variations of Net Energy



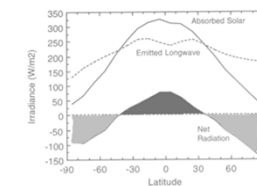
(from *Meteorology: Understanding the Atmosphere*)

- ❑ Polarward heat flux is needed to transport radiation energy from the tropics to higher latitudes.

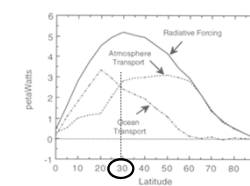
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Polarward Energy Transport

Annual-Mean Radiative Energy



Polarward Heat Flux



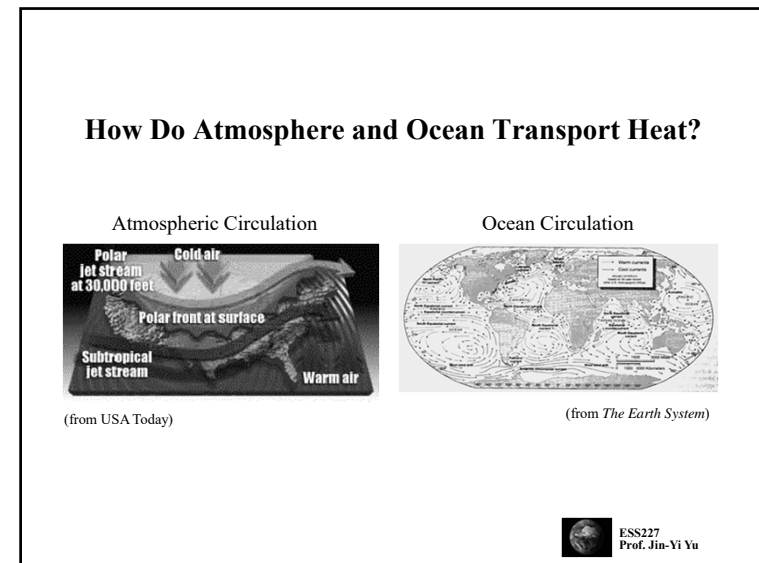
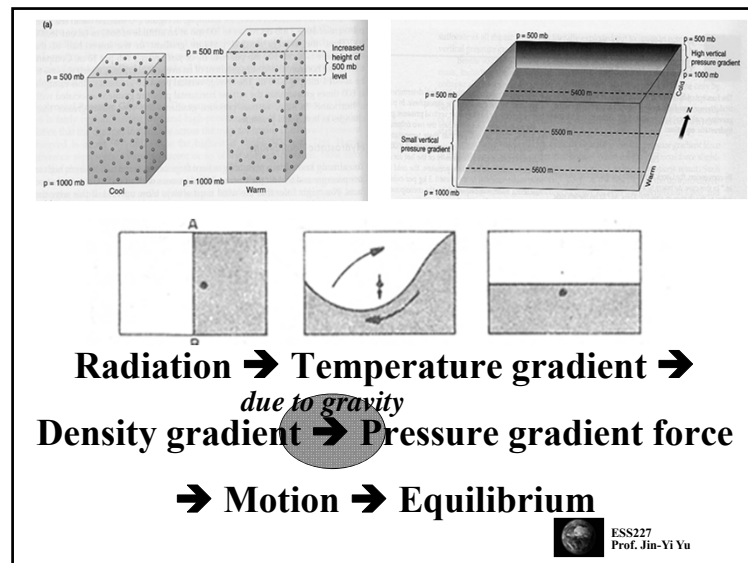
Polarward heat flux is needed to transport radiative energy from the tropics to higher latitudes

The atmosphere dominates the polarward heat transport at middle and high latitudes. The ocean dominates the transport at lower latitudes.

(1 petawatts = 10^{15} W)

(figures from *Global Physical Climatology*)

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Geophys. Fluid Motion and Global Energy Balance

- ❑ Vertical temperature gradients
 - \rightarrow Convection occurs that tries to reduce the vertical gradients
 - \rightarrow Vertical variation of air density (i.e., *stratification*)
- ❑ Horizontal temperature gradients
 - \rightarrow Fluid motion takes place to reduce the gradients
 - \rightarrow The motion (i.e., the *adjustment*) takes place in a *rotating and stratified system*.

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