Geophysics Fluid Dynamics (ESS228)

Course Time

Lectures: Tu, Th 09:30-10:50 Discussion: 3315 Croul Hall

Text Book

J. R. Holton, "*An introduction to Dynamic Meteorology*", Academic Press (Ch. 1, 2, 3, 4, 6, 8, 11).

Adrian E. Gill, "*Atmosphere-Ocean Dynamics*", Academic Press (Ch. 5, 6, 7, 8, 9, 10, 11, 12).

Grade

Homework (30%), Midterm (35%), Final (35%)

Homework

Issued and due every Thursday





SYLLABUS

Week 1	1/10 & 1/12	Introduction and Review of Mathematical Tools Mathematical tools and estimating with scale Fundamental and apparent forces
Week 2	1/17 & 1/19	Basic Conservation Laws Equations of motion, thermodynamic energy equation, Continuity equation
Week 3	1/24 & 1/26	Applications of the Equations of Motion Balanced (geostrophic, inertial, cyclostrophic, gradient) flows Thermal wind balance
Week 4	1/31 & 2/2	Circulation, Vorticity, and Divergence The Circulation theorem Vorticity and potential vorticity
Week 5	217 & 219	Waves in the Atmosphere Perturbation method Gravity wave, Rossby wave, Kelvin wave
Midterm	2/14	
Week 6	2/16	Adjustment Under Gravity In a non-rotating system
Week 7	2/21 & 2/23	Adjustment Under Gravity In a density-stratified fluid Effect of rotation
Week 8	2/28 & 3/1	Midlatitude Dynamics: Baroclinic Instabilities Concept of normal mode Continuously stratified atmosphere Energetics of baroclinic waves
Week 9	3/6 & 3/8	Ocean Circulation Wind-driven circulation Western Boundary currents
Week 10	3/13 & 3/15	Tropical Dynamics Scale analysis of large-scale tropical motions Equatorial wave theory
Final	TBD	en ê



ESS228: GEOPHYS FLUID DYNAMICS

Dynamics and Kinematics

- <u>Kinematics</u>: The term **kinematics** means *motion*. Kinematics is the study of motion without regard for the cause.
- <u>Dynamics</u>: On the other hand, **dynamics** is the study of the *causes* of motion.

This course discusses the physical laws that govern atmosphere/ocean motions.



Geophysical Fluid Dynamics

the study of the *causes* of motion

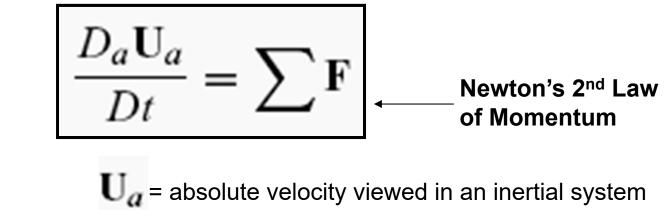
causes: solar radiation competes with gravity

distributions of T and motions

governed by conservation laws

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Newton's 2nd Law of Motion



 $\frac{D_a \mathbf{U}_a}{Dt} = \text{rate of change of } \mathbf{U}_a \text{ following the motion in an inertial system}$

• The conservation law for momentum (Newton's second law of motion) relates the rate of change of the absolute momentum following the motion in an inertial reference frame to the sum of the forces acting on the fluid.



Conservation of Mass

 The mathematical relationship that expresses conservation of mass for a fluid is called the *continuity equation*.

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \boldsymbol{\cdot} (\rho \mathbf{U}) = \mathbf{0}$$

(mass divergence form)

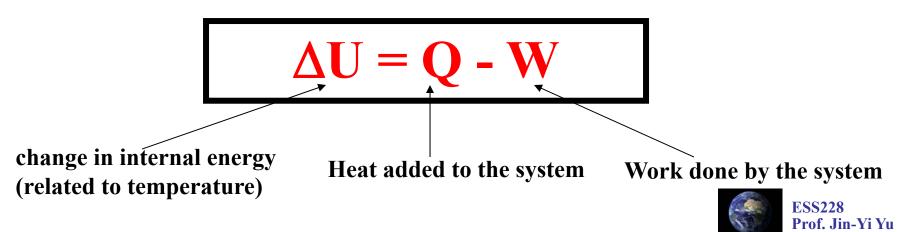
$$\frac{1}{\rho} \frac{D\rho}{Dt} + \nabla \cdot \mathbf{U} = 0$$

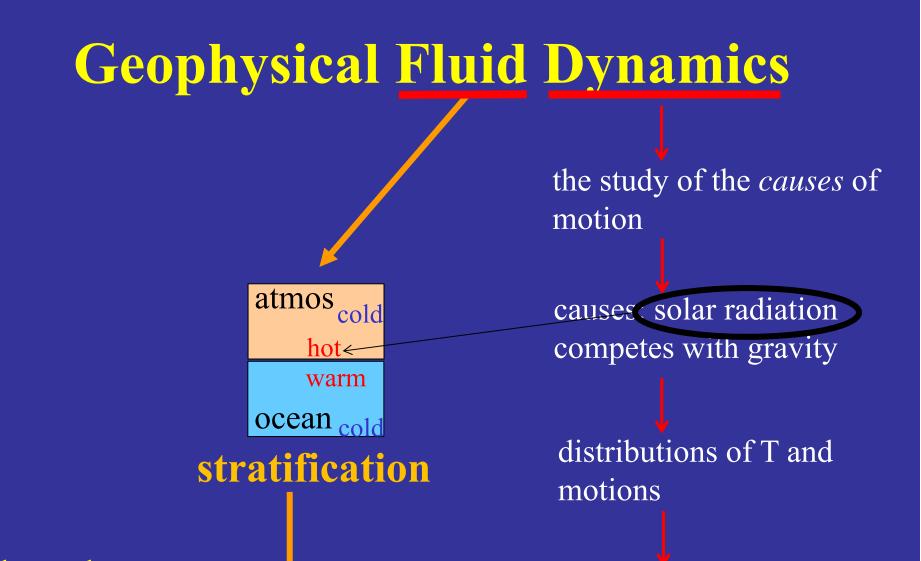
(velocity divergence form)



The First Law of Thermodynamics

- This law states that (1) heat is a form of energy that (2) its conversion into other forms of energy is such that total energy is conserved.
- The change in the internal energy of a system is equal to the heat added to the system minus the work down by the system:





understand atmos/ocean ← motions

expressed in mathematic forms laws and solve the equations



governed by conservation

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Stratification

Fluid that is less dense than its surroundings tends to rise until it has the same density as its surroundings. It will tend to become stratified.



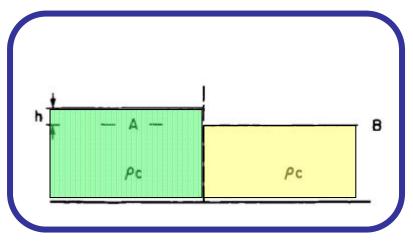
Adjustment Under Gravity in a Non-Rotating System

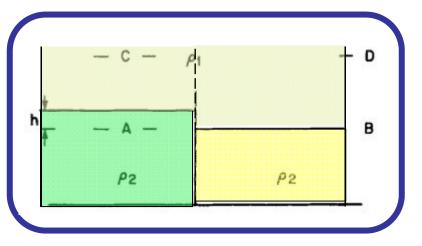
External Gravity Waves

adjustment of a homogeneous fluid with a free surface

Internal Gravity Waves

adjustment of a density-stratified fluid









Earth's rotation

atmos_{cold} hot← warm ocean_{cold}

stratification

the study of the *causes* of motion

causes solar radiation competes with gravity

distributions of T and motions

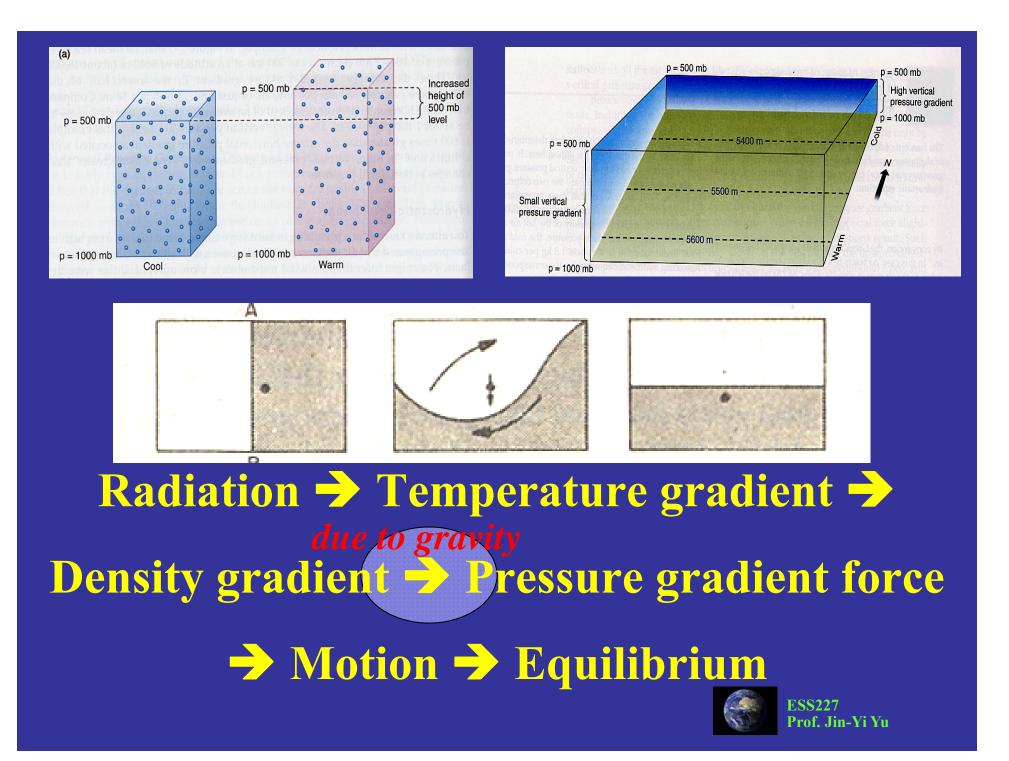
governed by conservation

understand atmos/ocean < motions

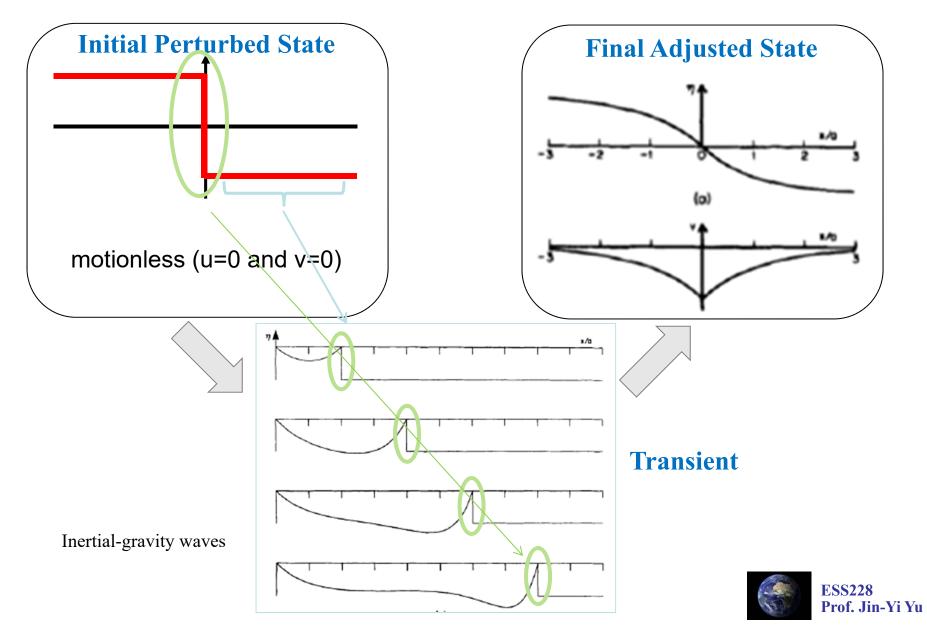
expressed in mathematic forms laws and solve the equations



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An Example of Geostrophic Adjustment



What Are the Issues?

- □ The fundamental aim is to understand the circulations of the atmosphere and ocean and the observed distributions of physical quantities such as temperature.
- The temperature distribution can be viewed as the result of a "competition" between the *sun, which tries to warm the tropics more than the poles* (and so create horizontal contrasts), and *gravity*, *which tries to remove horizontal* contrasts and arrange for warmer fluid to overlie colder fluid.
- This "competition" is complicated by such *effects as the rotation of the earth*, the variation of the angle between gravity and the rotation axis (the beta effect), and *contrasts between the properties of air and water*.



Atmosphere-Ocean Dynamics

Chapter Three Properties of a Fluid at Rest

Chapter Four Equations Satisfied by a Moving Fluid

Chapter Five Adjustment under Gravity in a Nonrotating System

Chapter Six Adjustment under Gravity of a Density-Stratified Fluid

Chapter Seven Effects of Rotation

Chapter Eight Gravity Waves in a Rotating Fluid

Chapter Nine Forced Motion

Chapter Ten Effects of Side Boundaries

Chapter Eleven The Tropics

Chapter Twelve Mid-latitudes

Chapter Thirteen Instabilities, Fronts, and the General Circulation

Dynamic Meteorology



Atmosphere-Ocean Dynamics

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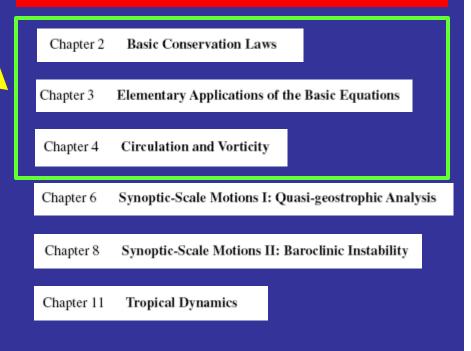
Chapter Ten Effects of Side Boundaries

Chapter Eleven The Tropics

Chapter Twelve Mid-latitudes

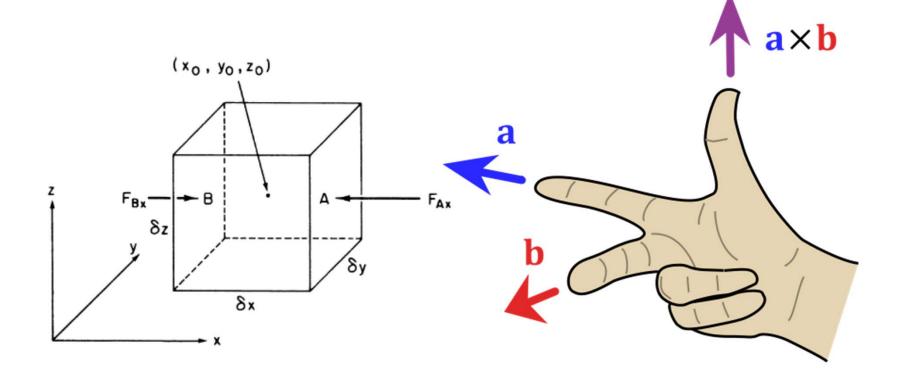
Chapter Thirteen Instabilities, Fronts, and the General Circulation

Dynamic Meteorology





Lecture 1: Introduction and Review



- Review of fundamental mathematical tools
- Fundamental and apparent forces



Basic Conservation Laws

Atmospheric motions are governed by three fundamental physical principles:

- conservation of mass (continuity equation)
- conservation of momentum (Newton's 2nd law of motion)
- conservation of energy (1st law of thermodynamics)

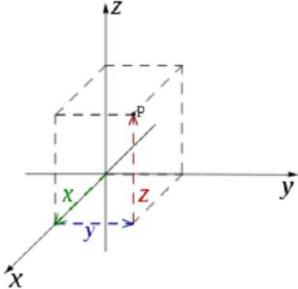
We need to develop mathematical formulas to describe these basic laws.



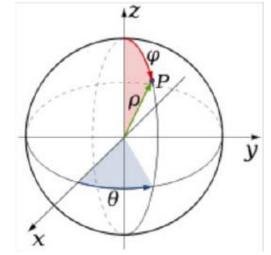
Coordinate System

A coordinate system is needed to describe the location in space.

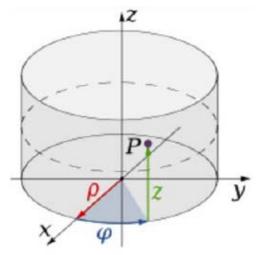
(1) Cartesian (x, y, z)



(2) Spherical (ρ, ϕ, θ)



The spherical coordinate system, showing a point, P and its coordinates, ρ , θ and φ (3) Cylindrical (ρ , ϕ , z)



A cylindrical coordinate system, showing radius, ρ , azimuth, φ and height, z.



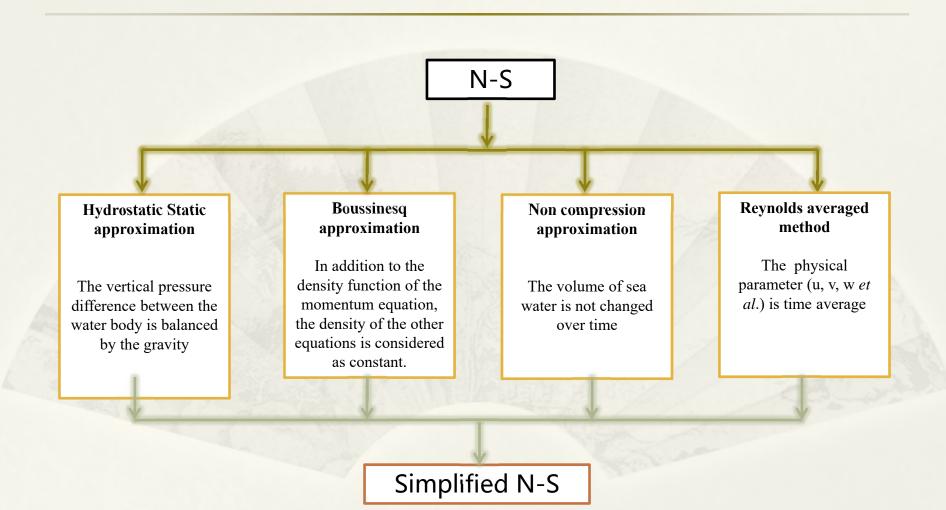
Basic question

N-S equation:

$\rho \left[\frac{du}{dt} - \frac{uv \tan \phi}{r} + \frac{uw}{r} \right] + 2\Omega \rho \left(w \cos \phi - v \sin \phi \right)$ $= -\frac{1}{r \cos \phi} \frac{\partial p}{\partial \lambda} + (\nabla \cdot \mathbf{T}) \cdot \hat{\lambda}$	
$\rho \left[\frac{dv}{dt} - \frac{u^2 \tan \phi}{r} + \frac{vw}{r} \right] + 2\Omega \rho u \sin \phi$ $= -\frac{1}{r} \frac{\partial p}{\partial \phi} + (\nabla \cdot \mathbf{T}) \cdot \hat{\phi}$	Momentum
$\rho \left[\frac{dw}{dt} - \frac{u^2 + v^2}{r} \right] - 2\Omega \rho u \cos \phi$	
$= -\frac{\partial p}{\partial r} - \rho g + (\nabla \cdot \mathbf{T}) \cdot \hat{r}$	
$\frac{d\rho}{dt} + \frac{\rho}{r\cos\phi} \left(\frac{\partial u}{\partial\lambda} + \frac{\partial(v\cos\phi)}{\partial\phi}\right) + \frac{\rho}{r^2} \frac{\partial(r^2w)}{\partial r} = 0$	Continuous
$\frac{dT}{dt} - \frac{\beta T}{\rho c_p} \frac{dp}{dt} = \frac{\nabla \cdot (\kappa \nabla T)}{\rho c_p} - \frac{\sigma}{\rho c_p}$	Thermal
	State
ho = ho(p,T) ,	State

Solving N-S equations is the basic problem in the numerical model of ocean circulation

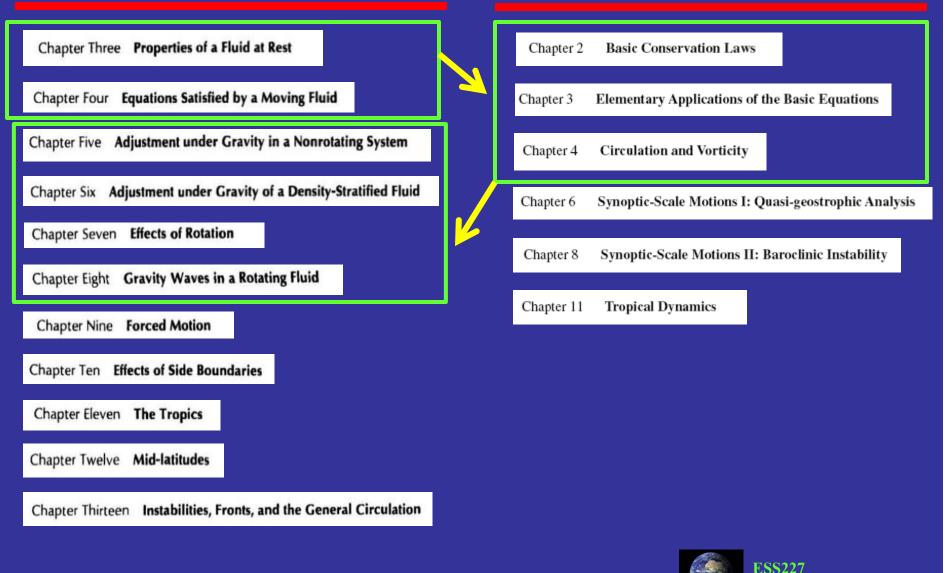
Simplification of N-S Equations



Atmosphere-Ocean Dynamics

Dynamic Meteorology

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Waves in the Atmosphere and Oceans

Restoring Force

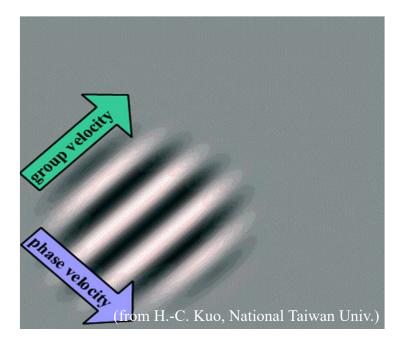
□Conservation of potential temperature in the presence of positive static stability
 → internal gravity waves
 □Conservation of potential vorticity in the presence of a mean gradient of potential vorticity → Rossby waves

- External gravity wave (Shallow-water gravity wave)
- Internal gravity (buoyancy) wave
- **Inertial-gravity wave**: Gravity waves that have a large enough wavelength to be affected by the earth's rotation.
- **Rossby Wave**: Wavy motions results from the conservation of potential vorticity.
- Kelvin wave: It is a wave in the ocean or atmosphere that balances the Coriolis force against a topographic boundary such as a coastline, or a waveguide such as the equator. Kelvin wave is non-dispersive.



Dispersion of Internal Gravity Waves

mean flow zonal wavenumber vertical wavenumber total wavenumber
$$\hat{v} \equiv v - \bar{u}k = \pm Nk/(k^2 + m^2)^{1/2} = \pm Nk/|\kappa|$$



□ In the atmosphere, internal gravity waves generated in the troposphere by cumulus convection, by flow over topography, and by other processes may propagate upward many scale heights into the middle atmosphere. \hat{v} is always smaller than N!!

Internal gravity waves can have any frequency between zero and a maximum value of N.

□ Phase velocity:

$$c_x = \hat{\nu}/k$$
 and , $c_z = \hat{\nu}/m$

Group velocity:

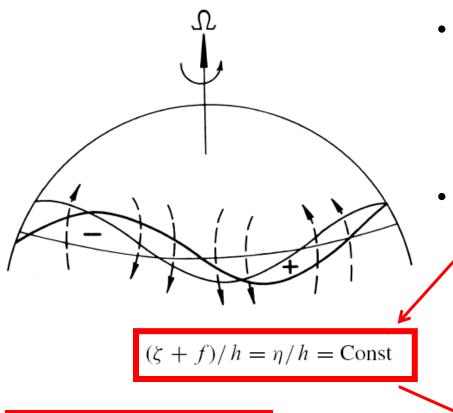
$$c_{gx} = \frac{\partial v}{\partial k} = \overline{u} \pm \frac{Nm^2}{\left(k^2 + m^2\right)^{3/2}}$$
$$a_{xy} = \frac{\partial v}{\partial v} = \pm \frac{(-Nkm)}{\left(-Nkm\right)}$$

$$c_{gz} = \frac{1}{\partial m} = \pm \frac{1}{\left(k^2 + m^2\right)^{3/2}}$$

□ Internal gravity waves thus have the remarkable property that group velocity is perpendicular to the direction of phase propagation.



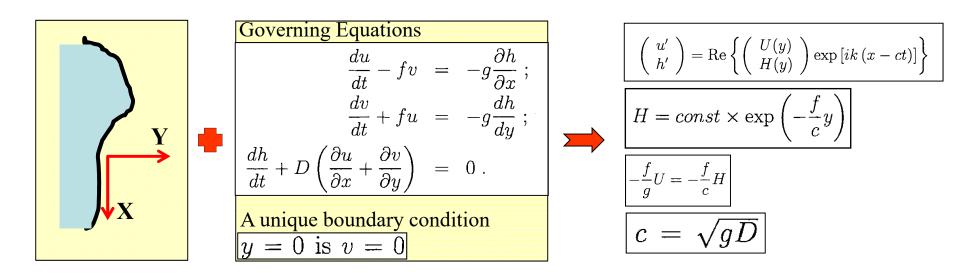
Rossby Wave



$$P \equiv (\zeta_{\theta} + f) \left(-g \frac{\partial \theta}{\partial p} \right) = \text{Const}$$

- The wave type that is of most importance for large-scale meteorological processes is the Rossby wave, or planetary wave.
- In an inviscid barotropic fluid of *constant depth* (where the divergence of the horizontal velocity must vanish), the Rossby wave is an absolute vorticity-conserving motion that owes its existence to the *variation of the Coriolis parameter* with latitude, the so-called β-effect.
- More generally, in a baroclinic atmosphere, the Rossby wave is a potential vorticity-conserving motion that owes its existence to the *isentropic gradient of potential vorticity*.

Kelvin Waves



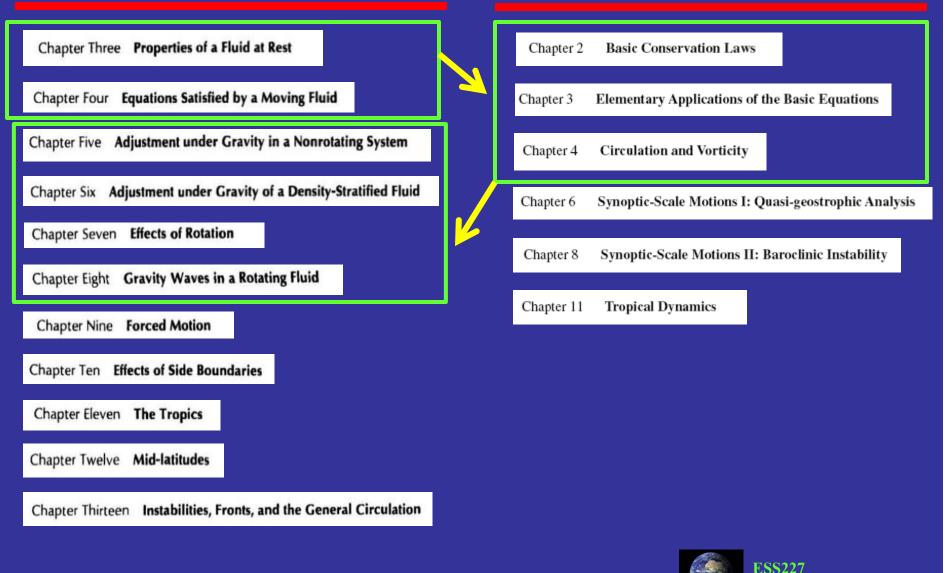
- A Kelvin wave is a type of low-frequency gravity wave in the ocean or atmosphere that balances the Earth's Coriolis force against a topographic boundary such as a coastline, or a waveguide such as the equator.
- Therefore, there are two types of Kelvin waves: coastal and equatorial.
- A feature of a Kelvin wave is that it is non-dispersive, i.e., the phase speed of the wave crests is equal to the group speed of the wave energy for all frequencies.



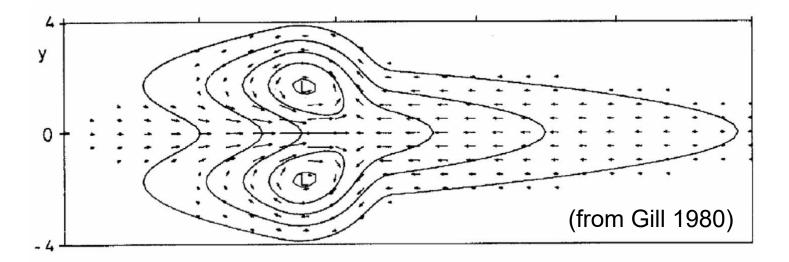
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Gill's Response to Symmetric Heating



- This response consists of a eastward-propagating Kelvin wave to the east of the symmetric heating and a westward-propagating Rossby wave of n=1 to the west.
- The Kelvin wave low-level easterlies to the east of the heating, while the Rossby wave produces low-level westerlies to the west.
- The easterlies are trapped to the equator due to the property of the Kelvin wave.
- The n=1 Rossby wave consists of two cyclones symmetric and straddling the equator.
- The meridional scale of this response is controlled by the equatorial Rossby radius, which is related to the β -effect and the stability and is typically of the order of 1000km.

Gill Type Response to Tropical Heating

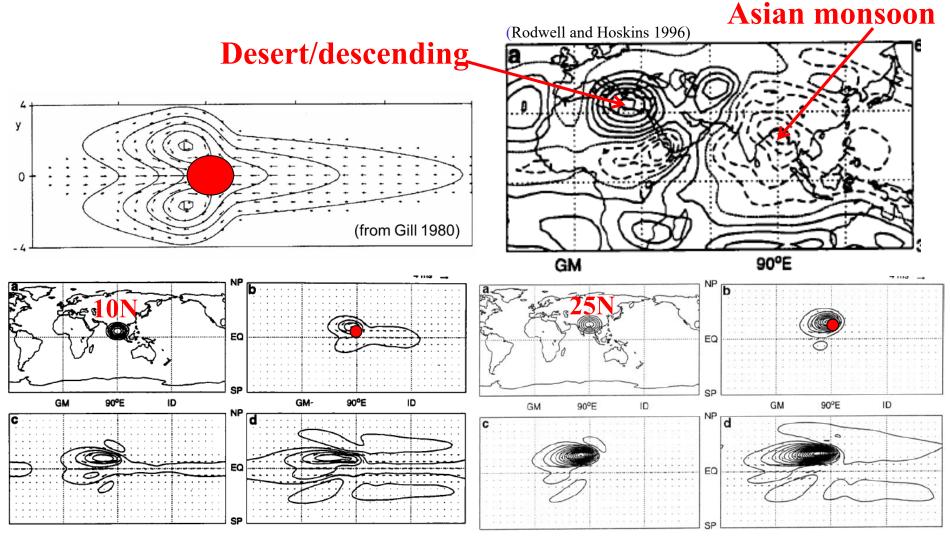


Figure 6. (a) Column-mean diabatic heating centred at 90°E, 10°N. The contour interval is 50 W m⁻²; the zero contour is not shown. (b), (c) and (d) show the corresponding perturbation surface pressure and 887 hPa horizontal winds for an integration linearized about a resting basic-state at (b) day 3, (c) day 7 and (d) day 11. The contour winds for an integration linearized about a resting basic-state at (b) day 3, (c) day 7 and (d) day 11. The contour interval is 1 hPa.

Figure 7. (a) Column-mean diabatic heating centred at 90°E, 25°N. The contour interval is 50 W m⁻²; the zero contour is not shown. (b), (c) and (d) show the corresponding perturbation surface pressure and 887 hPa horizontal interval is 1 hPa.

Gill-Type Response Mechanism

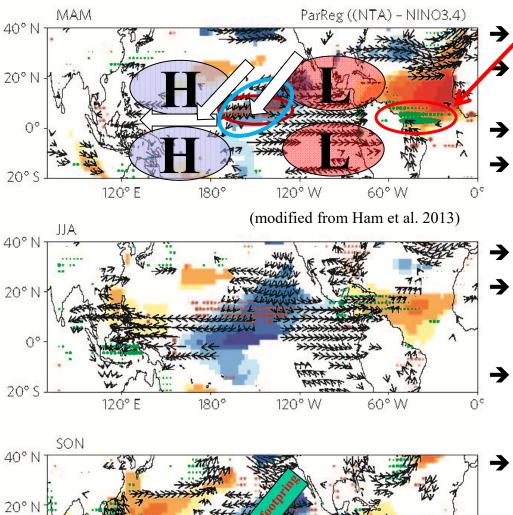
EFE WW

0°

60° W

120° W

180°



00

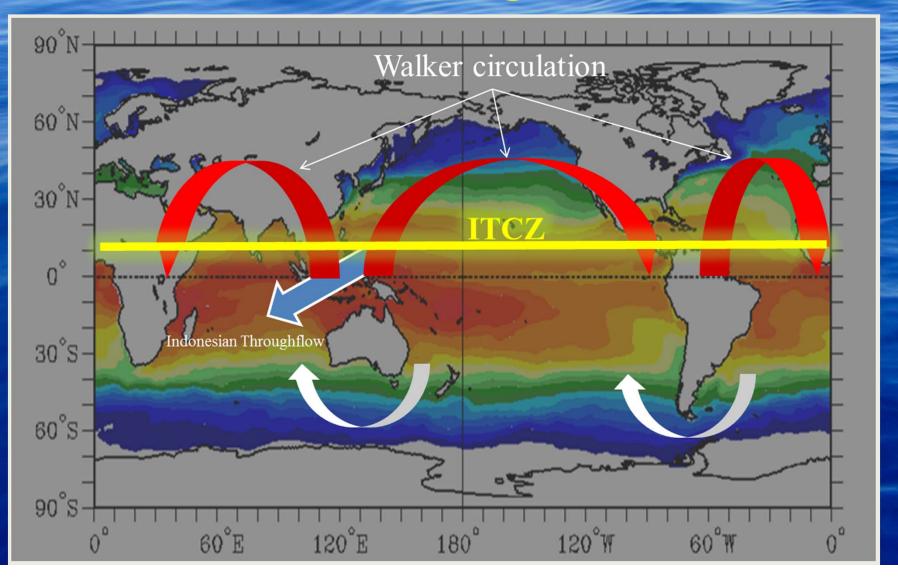
20° S

120° E

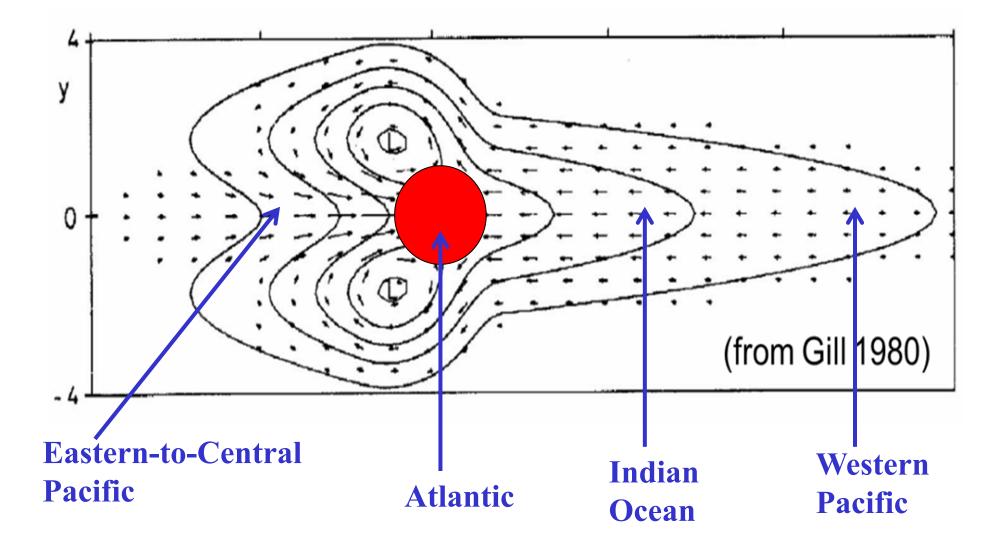
NTA warming in boreal spring

- → Enhances Atlantic ITCZ convection
 - Induces a low-level cyclonic atmospheric flow over the eastern Pacific (i.e., Gill type response)
 - Produces a northerly flow on its west flank
 - The northerly flow leads to surface cooling through the enhanced wind speed and cold/dry advection from higher latitudes
- ➔ Suppresses Pacific ITCZ convection
- Produces a low-level anticyclonic flow over the western Pacific during the following summer (i.e., Gill type response)
- This anticyclonic flow enhances the northerly flow at its eastern edge, which reinforces the negative precipitation anomaly.
- This coupling maintain and negative precipitation anomalies and generates easterly winds over the western equatorial Pacific
- → The winds cool the equatorial Pacific
- ➔ May trigger a Central-Pacific type of La Niña event the following winter.

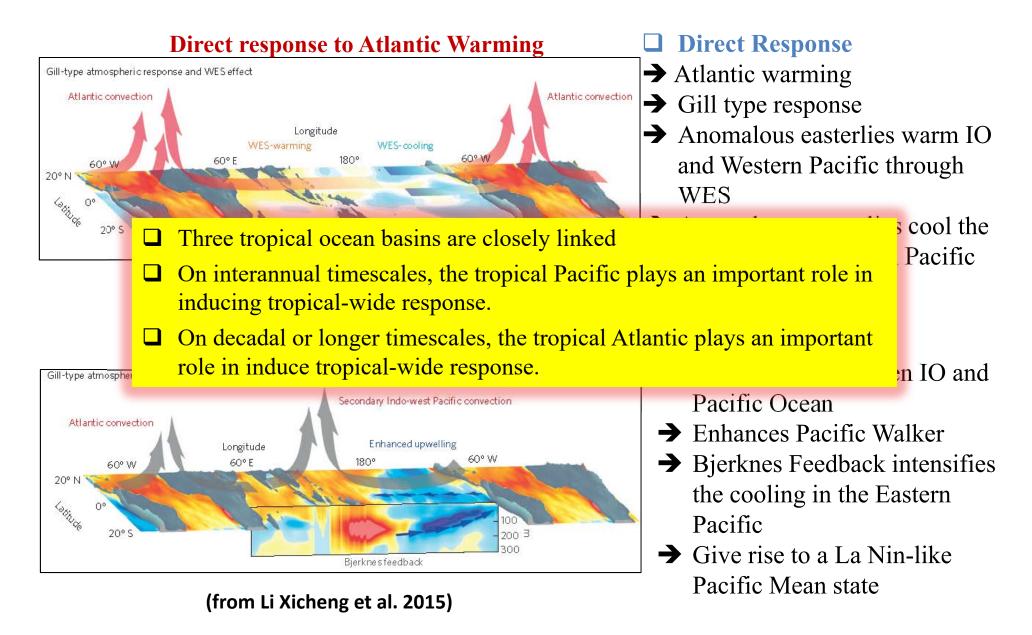
Inter-Basin Interactions (On Decadal or Longer Timescales)



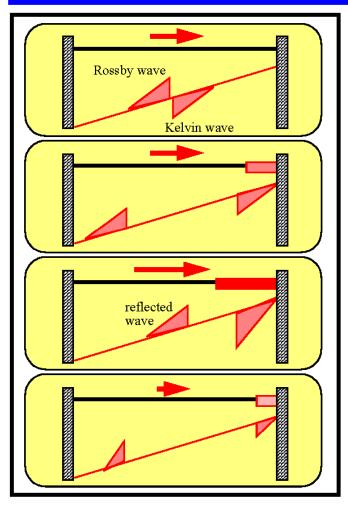
Gill Type Response to Tropical Heating



Trend: Atlantic Warming -> Pacific Cooling

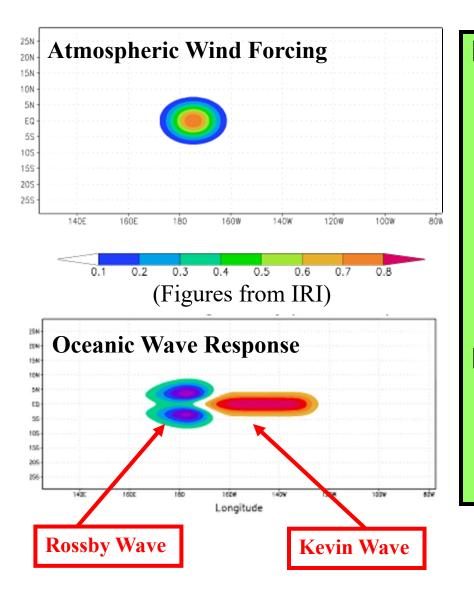


Delayed Oscillator Theory



- Wind forcing at the central Pacific: produces a downwelling Kevin wave propagating eastward and a upwelling Rossby wave propagating westward.
- wave propagation: the fast kelvin wave causes SST warming at the eastern basin, while slow Rossby wave is reflected at the western boundary.
- wave reflection: Rossby wave is reflected as a upwelling Kelvin wave and propagates back to the eastern basin to reverse the phase of the ENSO cycle.
- ENSO period: is determined by the propagation time of the waves.

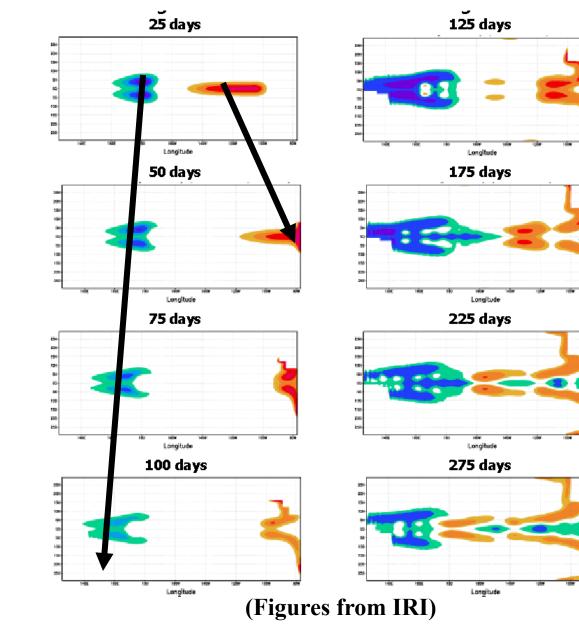
Delayed Oscillator: Wind Forcing



- The delayed oscillator suggested that oceanic Rossby and Kevin waves forced by atmospheric wind stress in the central Pacific provide the phase-transition mechanism (I.e. memory) for the ENSO cycle.
- The propagation and reflection of waves, together with local air-sea coupling, determine the period of the cycle.



Wave Propagation and Reflection



 It takes Kevin wave (phase speed = 2.9 m/s) about 70 days to cross the Pacific basin (17,760km).

It takes Rossby wave about 200 days (phase speed = 0.93 m/s) to cross the Pacific basin.





SYLLABUS

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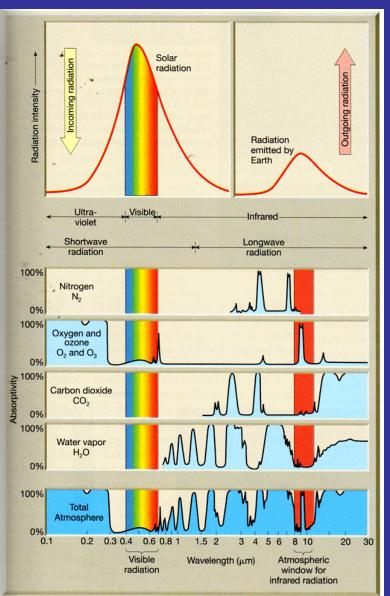
ESS228: GEOPHYS FLUID DYNAMICS

Chapter One How the Ocean-Atmosphere System Is Driven

- 1.1 Introduction
- 1.2 The Amount of Energy Received by the Earth
- 1.3 Radiative Equilibrium Models
- 1.4 The Greenhouse Effect
- 1.5 Effects of Convection
- 1.6 Effects of Horizontal Gradients
- 1.7 Variability in Radiative Driving of the Earth



Selective Absorption and Emission

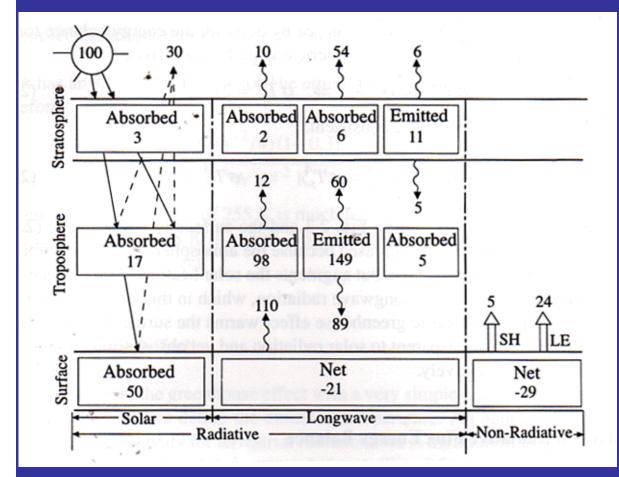


 □ The atmosphere is not a perfect blackbody, it absorbs some wavelength of radiation and is transparent to others (such as solar radiation). → Greenhouse effect.

- Objective that selectively absorbs radiation usually selectively emit radiation at the same wavelength.
- For example, water vapor and CO2 are strong absorbers of infrared radiation and poor absorbers of visible solar radiation.



Vertical Distribution of Energy



Incoming solar radiation

70% absorbed by Earth
 50% by Earth's surface
 20% by atmosphere

Outgoing terrestrial radiation

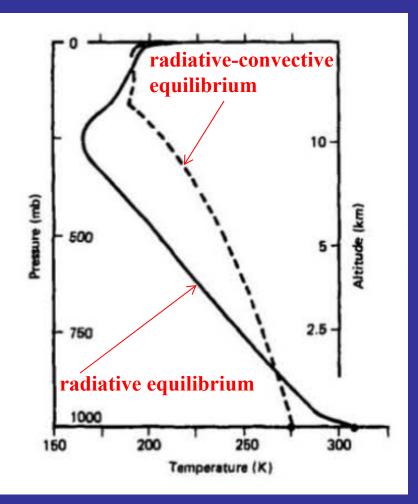
- 70 (units) back to space
 - 21% by surface
 - 49% by the atmosphere



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(from Global Physical Climatology)

Effect of Convection



Radiative Equilibrium: The temperature distribution that would be obtained based on the radiative energy balance in the absence of fluid motion.

Radiative-Convective Equilibrium: The temperature distribution that would be obtained based on a balance between radiative and convective effects.

 Whether or not convection will occur depends on the "lapse" rate, i.e., the rate at which the temperature of the atmosphere decreases with height.
 Convection will only occur when the lapse rate exceeds a certain value.



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Potential Temperature (\theta)

□ The potential temperature of an air parcel is defined as the the temperature the parcel would have if it were moved adiabatically from its existing pressure and temperature to a standard pressure P_0 (generally taken as 1000mb).

$$\Theta = T\left(\frac{P_0}{P}\right)^{\frac{R}{C_p}}$$

 θ = potential temperature T = original temperatureP = original pressure $P_0 = \text{standard pressure} = 1000 \text{ mb}$ $R = gas constant = R_d = 287 J deg^{-1} kg^{-1}$ $C_p = \text{specific heat} = 1004 \text{ J deg}^{-1} \text{ kg}^{-1}$ $R/C_{p} = 0.286$



Dry and Moist Adiabatic Lapse Rates

 \Box Dry adiabatic lapse rate is constant = 10°C/km.

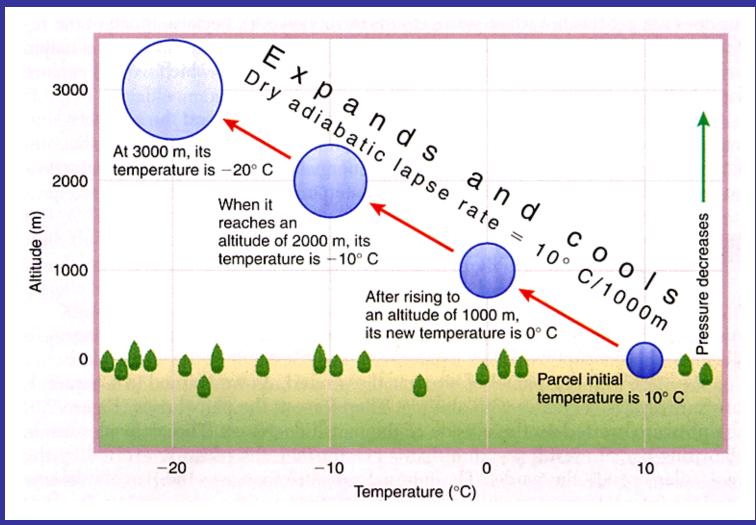
□ Moist adiabatic lapse rate is NOT a constant. It depends on the temperature of saturated air parcel.

□ The higher the air temperature, the smaller the moist adiabatic lapse rate.

→When warm, saturated air cools, it causes more condensation (and more latent heat release) than for cold, saturated air.



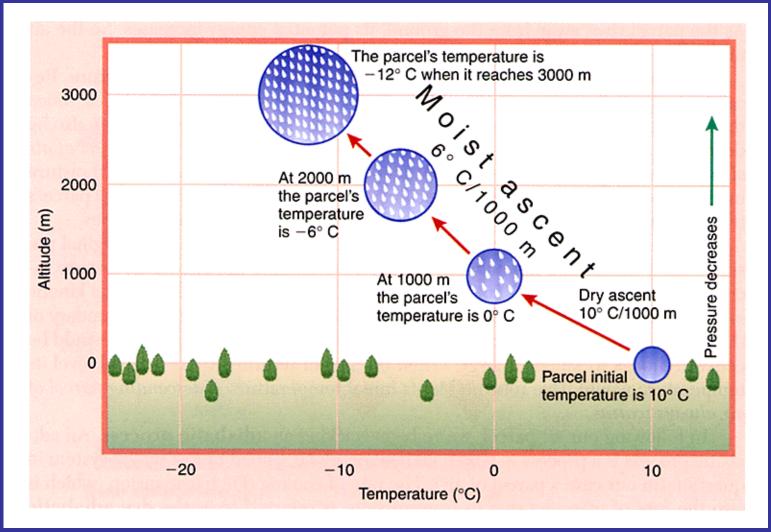
Dry Adiabatic Lapse Rate



(from Meteorology: Understanding the Atmosphere)



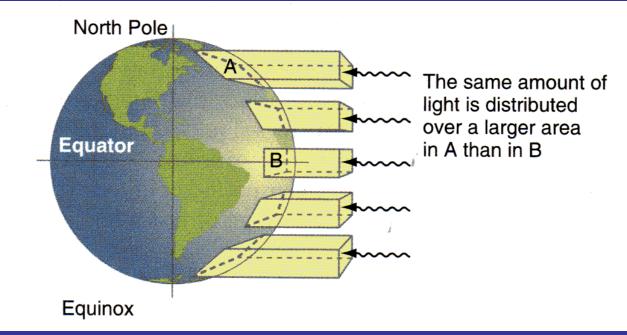
Moist Adiabatic Lapse Rate



(from *Meteorology: Understanding the Atmosphere*)



Zenith Angle and Insolation

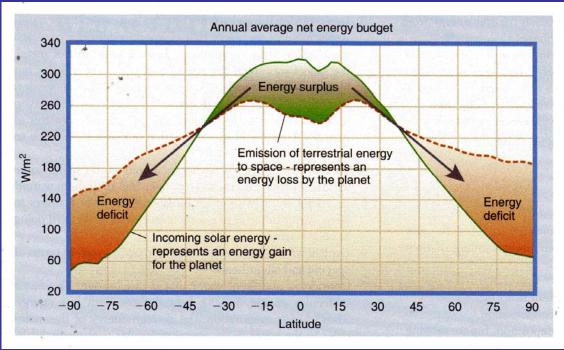


(from *Meteorology: Understanding the Atmosphere*)

□ The larger the solar zenith angle, the weaker the insolation, because the same amount of sunlight has to be spread over a larger area.



Latitudinal Variations of Net Energy



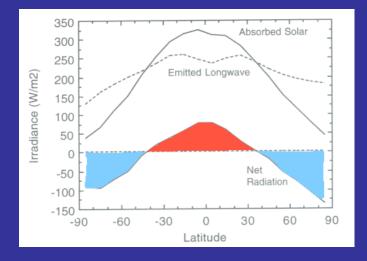
(from Meteorology: Understanding the Atmosphere)

Polarward heat flux is needed to transport radiation energy from the tropics to higher latitudes.



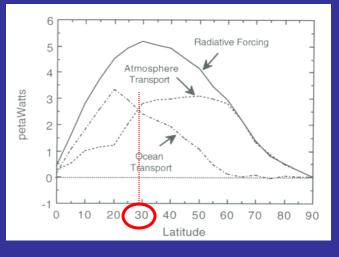
Polarward Energy Transport

Annual-Mean Radiative Energy



Polarward heat flux is needed to transport radiative energy from the tropics to higher latitudes

Polarward Heat Flux

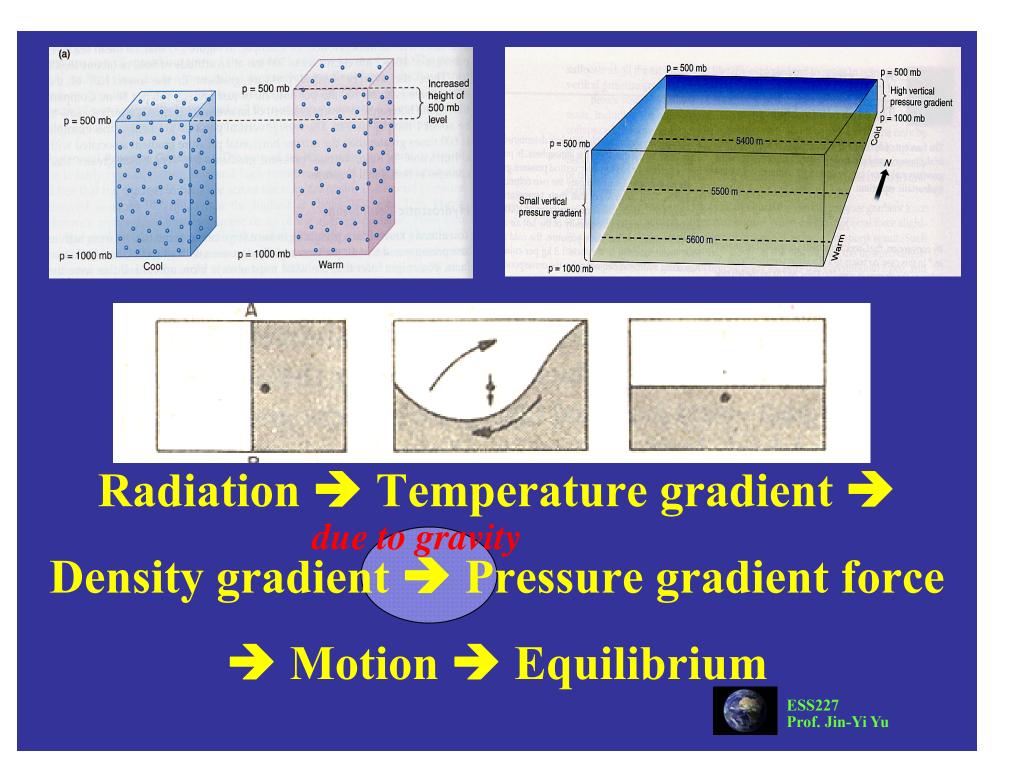


The atmosphere dominates the polarward heat transport at middle and high latitudes. The ocean dominates the transport at lower latitudes.

 $(1 \text{ petaWatts} = 10^{15} \text{ W})$

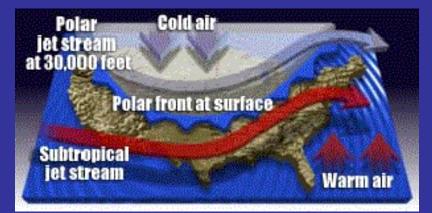


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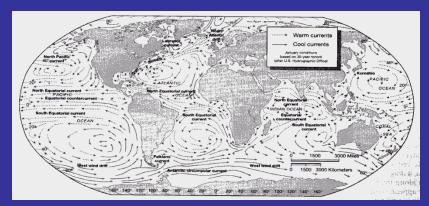
How Do Atmosphere and Ocean Transport Heat?

Atmospheric Circulation



(from USA Today)

Ocean Circulation



(from *The Earth System*)



Geophs. Fluid Motion and Global Energy Balance

□ Vertical temperature gradients

- Convection occurs that tries to reduce the vertical gradients
- → Vertical variation of air density (i.e., *stratification*)

Horizontal temperature gradients
 Fluid motion takes place to reduce the gradients
 The motion (i.e., the *adjustment*) *takes place in a rotating and stratified system*.

