Chapter 7: Forces and Force Balances

- Forces that Affect Atmospheric Motion
- Force Balance
- Geostrophic Balance and Jetstream
Forces that Affect Atmospheric Motion

- Newton’s second law of motion states that the rate of change of momentum (i.e., the acceleration) of an object, as measured relative to coordinates fixed in space, equals the sum of all the forces acting.

- For atmospheric motions of meteorological interest, the forces that are of primary concern are the pressure gradient force, the gravitational force, and friction. These are the fundamental forces.

- For a coordinate system rotating with the earth, Newton’s second law may still be applied provided that certain apparent forces, the centrifugal force and the Coriolis force, are included among the forces acting.
Pressure Gradient Force

- PG = (pressure difference) / distance
- Pressure gradient force goes from high pressure to low pressure.
- Closely spaced isobars on a weather map indicate steep pressure gradient.

(from Meteorology Today)
Examples of Pressure Gradient

Hurricane Andrew, 1992

Extratropical Cyclone

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Courtesy of American Meteorological Society
Pressure Gradients

- Pressure Gradients
  - The pressure gradient force initiates movement of atmospheric mass, wind, from areas of higher to areas of lower pressure

- Horizontal Pressure Gradients
  - Typically only small gradients exist across large spatial scales (1mb/100km)
  - Smaller scale weather features, such as hurricanes and tornadoes, display larger pressure gradients across small areas (1mb/6km)

- Vertical Pressure Gradients
  - *Average vertical pressure gradients are usually greater than extreme examples of horizontal pressure gradients* as pressure always decreases with altitude (1mb/10m)
Balance of Force in the Vertical: Hydrostatic Balance

Vertical pressure force = Gravitational force

\[- (dP) x (dA) = \rho x (dz) x (dA) x g\]

\[dP = -\rho g dz\]

Since \(P = \rho RT\),

\[dP = -P/RT x gdz\]
\[dP/P = -g/RT x dz\]

\[P = P_s \exp(-gz/RT)\]

(from Climate System Modeling)
Gravitational Force

- Newton’s law of universal gravitation states that any two elements of mass in the universe attract each other with a force proportional to their masses and inversely proportional to the square of the distance separating them.

- Thus, if the earth is designated as mass $M$ and $m$ is a mass element of the atmosphere, then the force per unit mass exerted on the atmosphere by the gravitational attraction of the earth is

  \[ \frac{F_g}{m} = g^* = -\frac{G M}{r^2} \left( \frac{r}{r} \right) \]

  \[ r = a + z \quad (a: \text{ earth radius}; \ z: \text{ height above surface}) \]

  \[ g^* = \frac{g_0^*}{(1 + z/a)^2} \]

  where $g_0^* = -(GM/a^2)(r/r)$ is the gravitational force at mean sea level.

For meteorological applications,

\[ z \ll a \quad \Rightarrow \quad g^* = g_0^* \]
Frictional Force

- Frictional force (drag) is strongest near the Earth’s surface and decreases rapidly with height.

- The atmospheric layer in which frictional force is important is called the boundary layer, whose depth can vary from a few hundred meters to a few thousand meters.

- There are three sources to generate turbulence eddies to give rise to the frictional force: (1) mechanical turbulence (airs encounter surface roughness), (2) thermal turbulence (air near Earth’s surface get heated), and (3) wind-shear induced turbulence.
Example on a Merry-Go-Around

(A) Direction of rotation

(B) True path

Apparent path seen by riders

© Kendall/Hunt Publishing
Coriolis Force – Conservation of Angular Momentum

Absolute angular momentum at A = Absolute angular momentum at B

\[ U_A \times (\text{radius at A}) = U_B \times (\text{radius at B}) \]

\[ U_A \times (\text{Earth’s Radius} \times \cos(\text{latitude at A})) = U_B \times (\text{Earth’s Radius} \times \cos(\text{latitude at B})) \]

\[ U_A = U_B \times \left[ \cos(\text{latitude at B}) / \cos(\text{latitude at A}) \right] \]

\[ U_A > U_B \]

⇒ A northward motion starting at A will arrive to the east of B due to the conservation of angular momentum

⇒ An apparent force (Coriolis Force) = \( f \cdot V \), where \( f = 2 \Omega \times \sin(\text{lat}) \)
where \( \Omega = 7.292 \times 10^{-5} \text{ rad s}^{-1} \)

⇒ To the right (left) of the motion in Northern (Southern) Hemisphere

(from The Earth System)
First, Point A rotates faster than Point B ($U_A > U_B$)

$U_A > U_B$

A northward motion starting at A will arrive to the east of B

It looks like there is a “force” pushing the northward motion toward right

This apparent force is called “Coriolis force”:

\[
\text{Coriolis Force} = f \, V
\]

where $f = 2\Omega \sin(\text{lat})$ and $\Omega = 7.292 \times 10^{-5} \text{ rad s}^{-1}$

(from *The Earth System*)
Coriolis Force

- Coriolis force causes the wind to deflect to the right of its intent path in the Northern Hemisphere and to the left in the Southern Hemisphere.

- The magnitude of Coriolis force depends on (1) the rotation of the Earth, (2) the speed of the moving object, and (3) its latitudinal location.

- The stronger the speed (such as wind speed), the stronger the Coriolis force.

- The higher the latitude, the stronger the Coriolis force.

- The Coriolis force is zero at the equator.

- Coriolis force is one major factor that determine weather pattern.
Coriolis Force Change with latitudes

(from The Atmosphere)
How Does Coriolis Force Affect Wind Motion?

(from Weather & Climate)
Geostrophic Balance

- By doing scale analysis, it has been shown that large-scale and synoptic-scale weather systems are in geostrophic balance.

- Geostrophic winds always follow the constant pressure lines (isobar). Therefore, we can figure out flow motion by looking at the pressure distribution.
Scales of Motions in the Atmosphere

(from Meteorology Today by C. Donald Ahrens © 1994 West Publishing Company)
Example: Winds and Height on 500mb
Frictional Effect on Surface Flow

- Surface friction force slows down the geostrophic flow.
- The flow turns into (out of) the low (high) pressure sides.
- Convergence (divergence) is produced with the flow.
Surface Friction

- Friction Force = $c \times V$
  
  $c =$ friction coefficient
  
  $V =$ wind speed
Surface Geostrophic Flow

Cyclonic Flow

Anticyclonic Flow

(figures from *Weather & Climate*)
(from *The Atmosphere*)
Surface High and Low Pressure Systems

(from *The Atmosphere*)
Super- and Sub-Geostrophic Wind

For high pressure system
- gradient wind > geostrophic wind
- supergeostropic.

For low pressure system
- gradient wind < geostrophic wind
- subgeostropic.

(from Meteorology: Understanding the Atmosphere)
Gradient Wind Balance

- The three-way balance of horizontal pressure gradient, Coriolis force, and the centrifugal force is called the **gradient wind balance**.

- The gradient wind is an excellent approximation to the actual wind observed *above* the Earth’s surface, especially at the middle latitudes.
• Upper tropospheric flows are characterized by trough (low pressure; isobars dip southward) and ridge (high pressure; isobars bulge northward).

• The winds are in gradient wind balance at the bases of the trough and ridge and are slower and faster, respectively, than the geostrophic winds.

• Therefore, convergence and divergence are created at different parts of the flow patterns, which contribute to the development of the low and high systems.
• The jetstream is a narrow band of strong winds that encircles the Earth in the mid-latitude.

• The band of strongest winds is typically 300 to 500 km wide and can extend from near the tropopause to about 500mb.

• The jetstream typically follows a wavelike pattern.

• The fastest jetstreams are the **Polar Front Jet**, commonly known as the Polar Jet, and the **Subtropical Jet** (STJ), which flow from west to east.
Two Jet Streams

- **Subtropical Jet**
  Located at the higher-latitude end of the Hadley Cell. The jet obtain its maximum wind speed (westerly) due the conservation of angular momentum.

- **Polar Jet**
  Located at the thermal boundary between the tropical warm air and the polar cold air. The jet obtain its maximum wind speed (westerly) due the latitudinal thermal gradient (thermal wind relation).
Vertical Structure of Jetstream

Vertical Cross Section of Temperature (°C) and Zonal Wind (m/s) at 180°E Longitude
January Climatology NCEP/NCAR Reanalysis (1981-2010)

(from Riehl (1962), Palmen and Newton (1969))
Polar jetstreams are found at latitudes from 30 to 70° and between 300 and 200 hPa pressure surfaces (about 7.5–11 km above sea level).

The Polar Jet is strongest during winter when it occasionally migrates to tropical latitudes and merges with the subtropical jet.
Jet Streaks

- The position and orientation as well as the strength and continuity of the Polar Jet governs weather events on time scales from a day to weeks.

- Local maxima, called jet streaks, occur along the jet, especially where the Polar Jet merges with the STJ or in regions of strong temperature gradients.

- Jet streaks are usually identified as oval-shaped maxima in the 300-200 hPa isotach analysis.
Jet Streaks

- When the jetstream is zonal, nearly directly west to east, short-wave disturbances and jet streaks move quickly along the jet track.

- In contrast, when the jetstream pattern is meridional, upper-level wave troughs and ridges are common, warm air flows poleward, and cold air flows equatorward, and the movement of disturbances is usually slower.

- Strong temperature gradients develop over limited longitude ranges and result in low pressure systems (cyclones) and high pressure systems (anticyclones) which cause persistent wet and dry conditions respectively.
During winter, the STJ is nearly continuous in both hemispheres and can attain wind speeds of 75 to 100 m s\(^{-1}\).

In the Northern Hemisphere, the jet exhibits a quasi-stationary 3-wave pattern with ridges and maximum wind speeds occurring over the southeastern United States, the Mediterranean Sea, and the northwest Pacific, which has the strongest winds. The troughs are usually located over the central Pacific Ocean, the central Atlantic Ocean, and between the Arabian Sea and India.

The mean position of the STJ in the Northern Hemisphere during winter is approximately 27.5°N, ranging from 20° to 35°N.

The STJ exists all year in the southern hemisphere. However, it is intermittent in the northern hemisphere during summer when it migrates north, with the mean position being close to 40°N and average speed decreases to about 35 m s\(^{-1}\).
Subtropical Jet

- High wind speeds emanating from the east coasts of Asia and North America. The pattern is more zonal and jets are stronger over the southern hemisphere because of the smaller land mass.

- Departures from the mean position are more pronounced over the America-Atlantic sector than in the Africa-Asia sector.

- The STJ can be temporarily displaced when strong mid-latitude troughs extend into subtropical latitudes. When these displacements occur, the subtropical jet can merge with the polar front jet.

- The mean latitude of the southern hemispheric STJ is less variable, shifting from about 26°S in winter to about 32°S in summer.
In this case, the meridional jet stream pattern and the strong upper-level jet streak aided in the development of a surface cyclone to the east of Jamaica near 15 °N.
Jetstream and Front

Strongest Horizontal Pressure Gradient located at position of jetstream (J).

Warm air

Cold air
Hydrostatic Balance in the Vertical

- Vertical pressure force = gravitational force

\[- (dP) \times (dA) = \rho \times (dz) \times (dA) \times g\]

\[
dP = -\rho g dz
\]

\[
dP/dz = -\rho g
\]

*The hydrostatic balance!!*

(from Climate System Modeling)
Thermal Wind Relation

(from Weather & Climate)
Thermal Wind Equation

\[ \frac{\partial U}{\partial z} \propto \frac{\partial T}{\partial y} \]

- The vertical shear of zonal wind is related to the latitudinal gradient of temperature.
- Jet streams usually are formed above baroclinic zone (such as the polar front).