Atmospheric Dynamics, EarthSS 112/212
Mid-term, 7 February, 2002

Some constants: \(a=6370\ \text{km}, \ g=9.8\ \text{ms}^{-2}, \ c_p = 1004\text{Jkg}^{-1}\text{K}^{-1}, \ R = 287\text{Jkg}^{-1}\text{K}^{-1}, \ \Omega = 7.292 \cdot 10^{-5}\text{s}^{-1}\).

**I.** (112: 35%, 212: 30%)

a) Write down both components of the geostrophic wind in pressure coordinates. Write down the hydrostatic equation in pressure coordinates.

b) Using a), derive both components of the thermal wind (i.e., the change in both components of the geostrophic wind with pressure).

c) Express in your own words the physical meaning of b).

d) How does b) and therefore c) change if the atmosphere is barotropic?

e) Consider an atmospheric layer, stretching from 90kPa to 70kPa. The vertically averaged temperature in this layer decreases toward the northwest at the rate of \(4^\circ\text{C}/100\ \text{km}\). If the geostrophic wind at 70kPa is from the west at 25 m/s, what are both components of the geostrophic wind at 90kPa?
II. (112: 35%, 212: 40%)

Consider a steady symmetric hurricane in gradient wind balance at 30°N. The radial dependence of the azimuthal wind speed is \( V(r) = V_0 \left( \frac{r}{r_0} \right)^n \), where \( n, r_0 \) and \( V_0 \) are constants.

a) What is the circulation at radius \( r \)?

b) What is the vorticity at radius \( r \)?

Suppose the potential vorticity of each parcel of air is conserved as the hurricane migrates from 30°N to 45°N, the vortex is an incompressible fluid of depth \( H \) which is 15 km at 30°N and 12 km at 45°N.

c) What is the absolute vorticity for a parcel of air at latitude 30°N? What is the absolute vorticity for a parcel of air at latitude 45°N?

d-e) What is the velocity distribution \( (V(r)) \) when the symmetric hurricane reaches 45°N? [This part is not required for EarthSS 112, but will give extra credit]
III. (20%)

Consider the “simplest forecast model”, which expresses the conservation of absolute vorticity in two dimensional geostrophic flow on the β plane.

a) Write down the prognostic (forecast) equation as it would be solved. What conditions are required? (Initial conditions)

b) Derive the invertibility relation.

c) Describe the algorithm to solve this problem numerically. (What are the steps to making a forecast (as in a flow chart)?)

d) What would be an equivalent conservation law for three dimensional flow that would allow a similar forecast model only three dimensional?