Atmospheric Dynamics, EarthSS 112/212

Homework due February 19, 2002

Some constants: \( a = 6370 \text{ km} \), \( g = 9.8 \text{ m s}^{-2} \), \( c_p = 1004 \text{ J kg}^{-1} \text{ K}^{-1} \), \( R = 287 \text{ J kg}^{-1} \text{ K}^{-1} \), \( \Omega = 7.292 \times 10^{-5} \text{ rad s}^{-1} \).

I.

Consider a steady symmetric hurricane in gradient wind balance at \( 30^\circ \text{N} \). The radial dependence of the azimuthal wind speed is \( V(r) = V_0 \left( \frac{r}{r_0} \right)^n \), where \( n \), \( r_0 \) and \( V_0 \) are constants.

Typical values of horizontal velocity, vertical velocity, horizontal scale, vertical scale, pressure variation and density are in SI units: \( U \approx 50 \), \( W \approx 1 \), \( L \approx 10^3 \), \( H \approx 10^3 \), \( \Delta p \approx 40 \text{ mb} \), \( \rho_0 \approx 1 \).

a) Sketch the balance of forces for the hurricane.

b) What is the circulation at radius \( r \)?

c) What is the vorticity at radius \( r \)?

d) Define a Rossby number for this flow and find its magnitude?

Suppose the potential vorticity of each parcel of air is conserved as the hurricane migrates from \( 30^\circ \text{N} \) to \( 45^\circ \text{N} \), the vortex is an incompressible fluid of depth \( H \) which is 15 km at \( 30^\circ \text{N} \) and 12 km at \( 45^\circ \text{N} \).

e) What is the absolute vorticity for a parcel of air at latitude \( 30^\circ \text{N} \)? What is the absolute vorticity for a parcel of air at latitude \( 45^\circ \text{N} \)?

f) What is the velocity distribution as a function of radius \( (V(r)) \) when the symmetric hurricane reaches \( 45^\circ \text{N} \)?

II.

Consider the “simplest forecast model”, which expresses the conservation of absolute vorticity in two dimensional geostrophic flow on the \( \beta \) plane.

a) Write down the prognostic (forecast) equation as it would be solved. What conditions are required?

b) Derive the invertibility relation. What would be appropriate boundary conditions?

c) Describe the algorithm to solve this problem numerically. (What are the steps to making a forecast (as in a flow diagram)?)

d) What would be an equivalent conservation law for three dimensional flow that would allow a similar forecast model only three dimensional?