

Differential eddy diffusion of biogeochemical tracers

C. Pasquero

Geological and Planetary Sciences Division, California Institute of Technology, Pasadena, California, USA

Received 1 June 2005; revised 2 August 2005; accepted 4 August 2005; published 3 September 2005.

[1] The marine ecosystem dynamics in the subtropical regions is controlled by the availability of nutrients. Their supply depends on lateral transport from higher latitudes, which is affected by the characteristics of the oceanic mesoscale turbulence. In many cases turbulent transport is parameterized in terms of eddy diffusion. We here test the validity of this approach in a mesoscale turbulence model coupled to a simple ecosystem model. We show that the parameterization can be profitably used, provided that an effective eddy diffusion coefficient that depends on the reaction time scale of the transported tracers is used. If the diffusion coefficient valid for conservative tracers is used to express diffusion of nutrients and plankton, turbulent transport and biological productivity can be significantly overestimated. **Citation:** Pasquero, C. (2005), Differential eddy diffusion of biogeochemical tracers, *Geophys. Res. Lett.*, *32*, L17603, doi:10.1029/2005GL023662.

1. Introduction

[2] Photosynthetic activity in the ocean requires availability of nutrients in the surface layer. In upwelling regions, as subpolar gyres and coastal areas, supply of nutrients occurs via vertical transport from deep waters. Conversely, vertical transport is not efficient as nutrient supply into the large downwelling subtropical regions, where vertical velocities are predominantly directed downward and the thermocline that separates the nutrient poor surface mixed layer from the rich deep water is relatively deep. In those regions, lateral transport from high latitude upwelling waters is an important source of nutrients [Sarmiento *et al.*, 2004; Garçon *et al.*, 2001]. Oceanic mesoscale and submesoscale features, such as coherent eddies and frontal regions, affect both the lateral advection and the upwelling patterns responsible for nutrient input [Flierl and McGillicuddy, 2002]. A correct representation of their effects on ecosystem dynamics is thus necessary for obtaining reliable and consistent results from models [Doney *et al.*, 2004].

[3] The effects of coherent eddies on passive tracer dynamics have been subject to intense study (see, e.g., the review by Provenzale [1999]). Coherent vortices trap passive tracers for long times and limit fluid exchanges between the vortex cores and the surrounding turbulent background. The eddies have noticeable effects on the statistical properties of the system: Vortices are responsible for the emergence of non-Gaussian velocity distributions [Bracco *et al.*, 2000], and induce anomalous dispersion [Solomon *et al.*, 1993]. Parameterizations of particle dispersion that explicitly take into account the presence of the

vortices have been developed and tested [Pasquero *et al.*, 2001; Reynolds, 2002; Berloff and McWilliams, 2002; Veneziani *et al.*, 2005].

[4] A different question concerns the effects of the vortices on reactive tracers, and, in particular, the interaction between coherent vortices and marine ecosystem dynamics. The turbulent cascade to small scales is believed to generate plankton patchiness [Abraham, 1998; Lévy, 2003]. The secondary vertical circulation associated with mesoscale eddies has been shown to provide a localized source of nutrients [McGillicuddy and Robinson, 1997], and some of the consequences of the mesoscale fragmentation of nutrient input regions have been analyzed [Martin *et al.*, 2002; Pasquero *et al.*, 2005].

[5] Given the important role played by mesoscale structures, we here focus on their effects on lateral transport of nutrients and plankton. We explore whether some of the basic aspects of ecosystem dynamics in mesoscale turbulence can be reproduced when eddy diffusion is used to mimic turbulent transport. To this end, we use a simplified model of mesoscale turbulence based on the integration of the barotropic quasi-geostrophic equations, and a simple Nutrient-Phytoplankton-Zooplankton (NPZ) model to describe ecosystem dynamics.

2. Model Formulation

[6] The ecosystem model adopted here is a standard reaction-advection NPZ model [Fasham, 1993]:

$$\begin{cases} \frac{\partial N}{\partial t} + \mathbf{u} \cdot \nabla N = f(N, P, Z) = -S(\mathbf{x})(N - N_0) - \beta \frac{N}{k_N + N} P \\ \quad + \mu_N \left((1 - \gamma) \frac{a\epsilon P^2}{a + \epsilon P^2} Z + \mu_P P + \mu_Z Z^2 \right) \\ \frac{\partial P}{\partial t} + \mathbf{u} \cdot \nabla P = g(N, P, Z) = \beta \frac{N}{k_N + N} P - \frac{a\epsilon P^2}{a + \epsilon P^2} Z - \mu_P P \\ \frac{\partial Z}{\partial t} + \mathbf{u} \cdot \nabla Z = h(N, P, Z) = \gamma \frac{a\epsilon P^2}{a + \epsilon P^2} Z - \mu_Z Z^2. \end{cases} \quad (1)$$

The concentration fields of nutrient, N , phytoplankton, P , and zooplankton, Z , represent average concentrations in the euphotic layer, expressed in a common currency, e.g. amount of ammonium per volume of water. They are horizontally advected by the two-dimensional velocity field $\mathbf{u} \equiv (u, v)$, and undergo reactions as described by the terms on the right-hand-side (r.h.s.) of the equations. No explicit diffusion is included, and the semi-Lagrangian integration scheme adopted does not introduce numerical diffusion [Pasquero *et al.*, 2005]. This allows us to focus on the effects of pure advection: We aim at studying the transport of tracers induced by turbulence, in a domain characterized

by an inhomogeneous input of nutrients. The mixing induced by diffusion affects the ecosystem reaction rates in non trivial and interesting ways [Richards and Brentnall, 2005], that deserve further study.

[7] The terms on the r.h.s. of the equation for the nutrient represent respectively nutrient flux due to upwelling, conversion of nutrient into organic matter through phytoplankton activity, and regeneration of dead organic matter into nutrients. The regeneration efficiency, μ_N , is smaller than one and it represents the fact that not all biological substance becomes available as nutrient, as the fraction $(1 - \mu_N)$ is lost by sinking to deep water. The nutrient input is in the form of a relaxation flux which represents mixing between an infinite reservoir of nutrient-rich deep water, with nutrient concentration N_0 , and the mixed-layer water with nutrient concentration N . The relaxation time $1/S$ is a function of space (see Figure 2) and it measures the rate at which the nutrient relaxes to the value N_0 . With a restoring nutrient flux, the net nutrient input into the system depends on both the value of the relaxation rate, S , and the consumption by phytoplankton, which decreases the value of N with respect to N_0 [Martin et al., 2002; Pasquero et al., 2005].

[8] Phytoplankton dynamics is regulated by production, depending on available nutrients, through a Holling type II functional response, grazing by zooplankton, through a Holling type III functional response, and linear mortality. Zooplankton grow when phytoplankton are present (γ is the assimilation efficiency of the zooplankton), and die according to a quadratic mortality term used to close the system and parameterize the effects of higher trophic levels. Note that the present model does not account for light dependence of phytoplankton dynamics.

[9] The velocity field \mathbf{u} is taken to represent oceanic mesoscale turbulence. The model adopted here is the simple configuration known as barotropic turbulence, which is characterized by the presence of intense coherent vortices [McWilliams, 1984]. The velocity field is obtained by integrating the barotropic vorticity equation

$$\frac{\partial \zeta}{\partial t} + [\psi, \zeta] = F + D \quad (2)$$

where ψ is the stream function, $\zeta = \nabla^2 \psi$ is vorticity. The symbol $[a, b] = \frac{\partial a}{\partial x} \frac{\partial b}{\partial y} - \frac{\partial b}{\partial x} \frac{\partial a}{\partial y}$ is the two-dimensional Jacobian, and F and D are forcing and dissipation terms respectively. In the following numerical experiments, dissipation and forcing are kept as small as possible (consistent with the numerical resolution) so to have the highest possible Reynolds number, and they are balanced so that a statistically-stationary state is reached. The velocity field is given by the cross derivatives of the stream function, i.e., $\mathbf{u} \equiv (u, v) = \left(-\frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial x} \right)$. Further details on the model are given by Pasquero et al. [2005].

3. Relevant Time Scales

[10] Turbulent transport is often parameterized in terms of an eddy diffusivity, although diffusion is known to fail when non local effects are important [see, e.g., Lee et al., 1997; Sobel, 1999]. Taylor [1920] expressed the eddy diffusivity in a flow, κ , as twice the product of the turbulent

kinetic energy, E , by the Lagrangian decorrelation time-scale, T_L . In his quest for the derivation of the turbulent diffusion of heat, Taylor described the fluid motion by means of the corresponding Lagrangian trajectories, and assumed that the temperature of any particle remained constant during the motion. The derivation is therefore not directly applicable to the case of reactive tracers, whose concentration varies during the motion. Plumb [1979] derived an eddy diffusion parameterization for weakly non conserved tracers in a flow composed of small amplitude waves, and showed that the eddy diffusion coefficient in that case depends on the reaction time, T_R .

[11] The non dimensional number relevant for this study is the ratio between the advection time scale and the reaction time scale: the Damköhler number, $Da = T_L/T_R$. Recently, several studies have shown that the degree of patchiness in different models of plankton concentration is related to Da : Fast reactions do not allow for a high degree of homogenization, as the reactants are strongly consumed or produced before advection can efficiently stir them [Young et al., 2001; Mahadevan and Campbell, 2002; Neufeld et al., 2002].

[12] The Damköhler number for ecosystem variables in the upper ocean is usually order one or larger, as the typical decorrelation time is about a week [Veneziani et al., 2004] and plankton growth varies between one day (small phytoplankton) to a couple of weeks (zooplankton). In our model, we choose to scale the reaction timescale for each ecosystem variable with the inverse of its maximum growth rate observed during the simulations. The reaction timescale for, e.g., nutrient is $\min(N/f(N, P, Z))$. With the parameters used, the three ecosystem variables have the same reaction time scales, T_R . The study described in this paper has partially been repeated, leading to similar results, in the case the reaction time scale of Z was three times larger than the one of P .

4. Parameterization of Turbulent Advection

[13] The advection by the turbulent flow is here parameterized as eddy diffusion, so that system (1) is replaced by reaction-diffusion equations:

$$\begin{cases} \frac{\partial N}{\partial t} = f(N, P, Z) + \kappa \nabla^2 N \\ \frac{\partial P}{\partial t} = g(N, P, Z) + \kappa \nabla^2 P \\ \frac{\partial Z}{\partial t} = h(N, P, Z) + \kappa \nabla^2 Z \end{cases} \quad (3)$$

This approach is motivated by the fact that previous analysis of particle dispersion processes in numerical simulations of statistically stationary geostrophic turbulence have shown that the long-term single-particle dispersion is Brownian [Babiano et al., 1987; Pasquero et al., 2001], and a well-defined diffusion coefficient can be defined.

[14] The diffusion coefficient, κ , can be obtained by integrating the Lagrangian trajectories of an ensemble of fluid particles in the turbulent flow and, from these, by computing the time-dependent single-particle dispersion as:

$$D(t) = \frac{\langle [\mathbf{X}_j(t_0 + t) - \mathbf{X}_j(t_0)]^2 \rangle}{2t} \quad (4)$$

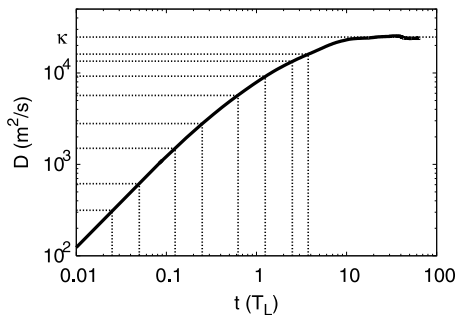


Figure 1. Diffusion function $D(t)$ obtained from the single-particle dispersion curve of an ensemble of 262,144 passive Lagrangian particles in barotropic turbulence. Dashed lines indicate the value of the reaction time scale for different ecosystem simulations, and the corresponding values of effective diffusivity. The turbulent flow is in a statistically stationary regime, characterized by the presence of many vortices with a long lifetime, which wander freely through the domain. Eddy kinetic energy is here $580 \text{ cm}^2/\text{s}^2$, and $T_L = 2.4$ days.

where $\mathbf{X}_j(t_0)$ is the position of the j -th particle at time t_0 , $\langle \cdot \rangle$ indicates the ensemble average over all particles and t is a time delay. For a statistically stationary flow the quantity $D(t)$ does not depend on the starting time t_0 . For a smooth flow, $D \propto t$ in the ballistic regime at small times, and $D(t) \rightarrow \kappa = 2ET_L$ in the Brownian regime at large times.

[15] The second parameterization that we test here consists in replacing κ in system (3) by an effective diffusivity, κ_{eff} , equal to the value of D at the reaction time scale T_R . As $D(t)$ increases monotonically with time, κ_{eff} is smaller than the asymptotic value κ (Figure 1). The rescaling of the diffusion coefficient is introduced as an extension of Plumb's theoretical result [Plumb, 1979] to systems of higher complexity. Its physical interpretation goes as follows. The spreading rate of a tracer in a smooth flow increases with time, until it saturates to κ when the Brownian regime is reached. If the tracer reacts before the Brownian regime is reached, it experiences an effective spreading rate smaller than κ . We consider that diffusion is

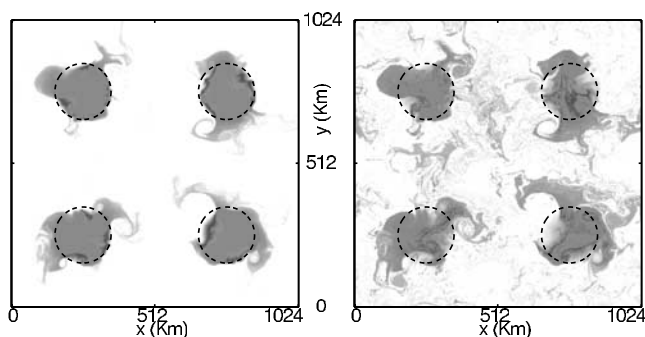


Figure 2. Distribution of primary productivity in the doubly-periodic domain, for $T_R/T_L = 0.12$ (left panel) and $T_R/T_L = 0.6$ (right panel). Dashed lines circumscribe the regions where nutrient is injected.

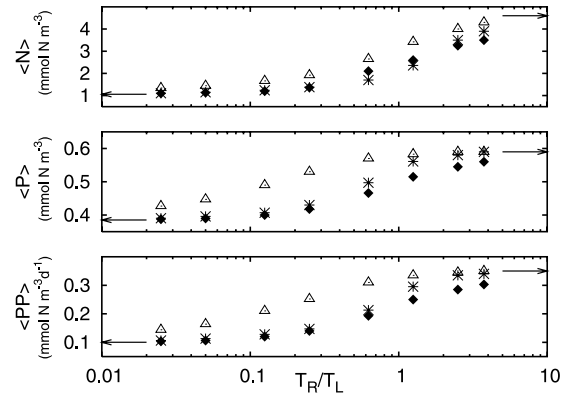


Figure 3. Domain averaged nutrient concentration (top panel), phytoplankton concentration (middle panel), and primary productivity (lower panel) for different values of the ratio T_R/T_L . Filled diamonds indicate results from the advection-reaction model (1); triangles refer to the eddy diffusion parameterization (3), when diffusivity is $\kappa = 2ET_L$; stars refer to the effective eddy diffusion parameterization, with $\kappa_{\text{eff}} = D(T_R)$. Arrows indicate asymptotic values for the case of no stirring (left) and of perfectly homogenized fields (right).

in this case regulated by the spreading rate that the tracer experiences at its reaction time scale.

5. Results of the Numerical Experiments

[16] Advection-reaction equations for the NPZ model are integrated using the barotropic turbulent flow as the advecting velocity field. Nutrient is injected in four circular regions of radius 100 km (Figure 2). Different simulations are performed, corresponding to different ratios between the reaction time and the Lagrangian decorrelation time, from 0.025 to 3.8. Two examples of the primary productivity, $PP = \beta \frac{N}{\kappa_N + N} P$, in the statistically stationary regime are shown in Figure 2. For short reaction times (left panel) primary productivity is concentrated in a region fairly close to the nutrient injection regions, as nutrients are consumed before being advected away. Larger spreading is obtained for longer reaction timescale (right panel).

[17] The mean values of nutrient concentration, phytoplankton concentration and primary productivity as function of T_R/T_L are represented in Figure 3 by filled diamonds. Primary productivity, nutrient and phytoplankton concentrations increase with the spreading and saturate at both ends. This effect has already been described in previous studies [Abraham *et al.*, 2000; Martin *et al.*, 2002; Pasquero *et al.*, 2005] and it is not discussed here.

[18] System (3) is then integrated, where pure diffusion is used to represent the effects of turbulent advection. In Figure 3 the results obtained using the diffusion coefficient κ are shown as triangles. This parameterization of the turbulent flow leads to overestimates of the mean nutrient concentration, phytoplankton concentration and primary productivity by as much as 70%. The obvious interpretation of those results is that the diffusion used to parameterize the turbulent advective flow is too strong, leading to excessive spreading of the tracers.

[19] The results obtained using κ_{eff} as eddy diffusion coefficient are shown in Figure 3 as stars. The parameter-

ization improves the results over the whole range of T_R/T_L , reducing deviations with respect to the eddy resolving case to less than 15%. Best agreement is obtained for small T_R . For $T_R \approx T_L$ the effective eddy diffusion parameterization does not exactly reproduce the results of the turbulent advection, as it still overestimates the mixing power of the turbulent flow. The discrepancies might be due to the presence of long-lived coherent structures that are known to reduce mixing, as their edges are strong barriers to transport and prevent water from their core to mix with external water [Provenzale, 1999; Martin *et al.*, 2002]. Those effects are not captured by the eddy diffusivity. Detailed understanding of the discrepancies will help to further improve the parameterization.

6. Conclusions

[20] Representation of turbulent sub-grid motion on transport as eddy diffusion, although not strictly justifiable, is often used in climate models. We test here how eddy diffusive transport in a NPZ model of the marine ecosystem reproduces the statistical results of an advection-reaction system where the advective component is given by a barotropic quasi-geostrophic turbulent model.

[21] When the biogeochemical tracers react on time scales shorter than the Lagrangian decorrelation time of the flow, they don't experience the constant spreading rate characteristic of the Brownian regime, and the diffusion law is not strictly applicable, even in absence of structures in the flow that create long range correlations.

[22] However, an effective eddy diffusivity is profitably introduced, as $\kappa_{eff} = D(T_R)$. As the function $D(t)$ is monotonically increasing with t , faster reacting tracers experience a smaller effective diffusivity. The use of the same value κ to represent turbulent mixing on conserved and non conserved tracers leads to excessive diffusion of reacting tracers.

[23] The issue is relevant for the study of the distribution of nutrients in the upper ocean, as phytoplankton bloom in a couple of days while Brownian like motion, if ever, is reached on a time scale of a week or more [Figueroa and Olson, 1994].

[24] Further studies will have to consider different reaction time scales for the different biogeochemical tracers, that have not been fully explored in the present letter.

[25] **Acknowledgments.** The author thanks M. Lévy, J. McWilliams, A. Provenzale, and T. Schneider for their important suggestions, and K. Richards for his insightful review. The author was supported by the Davidow's Discovery Fund.

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C. Pasquero, Geological and Planetary Sciences Division, California Institute of Technology, Pasadena, CA 91125, USA. (claudia@gps.caltech.edu)