ESS 130. Solutions to homework 6

1. (a) At the center of the Earth the tide producing force is zero, as the gravitational force towards the Moon and the centrifugal force exactly balance one another.
(b) At point X the centrifugal force is the same as anywhere else, $F_c = G\ M_E\ M_M/R_{EM}^2$. Here, G is the universal gravitational constant, M_E and M_M are mass of Earth and Moon respectively, R_{EM} is the distance between the center of the Earth and the center of the Moon. The gravitational force towards the Moon is smaller, as point X is further from the Moon than the center of the Earth, $F_g = G\ M_E\ M_M/(R_{EM}+a)^2$, where a is the Earth radius. The tide producing force is the difference

$$F_c - F_g = G\ M_E\ M_M(1/R_{EM}^2 - 1/(R_{EM}+a)^2) =$$

$$= G\ M_E\ M_M(R_{EM}^2 + a^2 + 2aR_{EM} - R_{EM}^2)/(R_{EM}^2(R_{EM}+a)^2) =$$

$$= G\ M_E\ M_M\ a(a+2R_{EM})/(R_{EM}^2(R_{EM}+a)^2) \simeq G\ M_E\ M_M\ 2\ a/R_{EM}^3$$

where the last passage is motivated by the fact that $R_{EM} \gg a$.
The force is directed away from the center of the Earth, in the vertical direction.
(c) Since the tide producing force at location X is in the vertical direction, it does not produce much motion in the water as it still is negligible compared to the gravitational force that acts along the same direction and points towards the center of the Earth.
(d) Calculate $c = 40,000km/24h50' = 450m/s$.
(e) Shallow water waves propagates at $c = \sqrt{g\ h}$, so

$$h = c^2/g = 450^2 m^2 s^{-2}/10m\ s^{-2} = 20km$$

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2. Here we have $\tau_{y,L} = -0.044kg\ m^{-1}\ s^{-2}$ and $\tau_{y,M} = -0.067kg\ m^{-1}\ s^{-2}$ (southerly winds implies that the x component of the wind stress is zero and the y component is negative).
(a) Wind stress curl is

$$\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y} = \frac{\tau_{y,L} - \tau_{y,M}}{20\ km} = 1.15 \cdot 10^{-6}\ N\ m^{-3}$$

Notice that wind stress curl is positive, as we could have guessed from the fact that this wind is injecting positive vorticity in the ocean.
(b) Ekman velocity averaged over Ekman depth is $u_E = \frac{\tau_y}{\rho\ f\ D}$, where ρ is water density, f is Coriolis parameter, and D is Ekman depth. Substituting for wind stress at L and M we obtain $u_{E,L} = -0.0088m/s$ and $u_{E,M} = -0.0134m/s$. Negative values indicate that Ekman transport is offshore (90^o to the right of the wind stress), and Ekman upwelling is expected to take place in this region.
(c) From conservation of mass, and considering that there is no alongshore flow, we have $\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$. Considering that at the surface vertical velocity is zero, we can write $\frac{\partial w}{\partial z} = \frac{w(50m)-0}{50m}$, from which

$$w(50m) = -50m\frac{(-0.0088 + 0.0134)m/s}{20,000m} = -1.15 \cdot 10^{-5}m/s = -1m/day.$$

The negative sign indicates that water is moving upward, as expected.

3. (a) Considering $L = 70 * 96km$ the width of the North Atlantic, we can write the longitudinally integrated North-South volume transport as

$$\frac{1}{\rho\beta}(curl\ \tau)\ L = \frac{-15 \cdot 10^{-8}\ kg\ m^{-2}s^{-2}70 \cdot 96,000m}{1000kg\ m^{-3}2 \cdot 10^{-11}\ m^{-1}\ s^{-1}} = -50{\cdot}10^6 m^3\ s^{-1} = -50Sv.$$

The negative sign indicates that transport is Southward, as expected from the fact that as the winds are injecting negative vorticity, to conserve angular momentum the water in the interior of the ocean must flow towards lower latitudes, where planetary vorticity is smaller.
(b) The value obtained is a crude estimate of the transport in the Gulf Stream. In fact, the southward flow in the interios of the basin must be compensated by a northward flow concentrated in a western boundary current, so that mass in conserved. The Gulf Stream transports the same amount of water, but in a narrow and strong current along the coast.